

Tracking Error

$$e_T \equiv r - y$$

$$e_T = r - [I + PK]^{-1} PKr - [I + PK]^{-1} P d_i \\ - [I + PK]^{-1} d_o + [I + PK]^{-1} PK n$$

$$= \left\{ I - [I + PK]^{-1} PK \right\} r + \text{other stuff}$$

↑ why to right?

$$= [I + PK]^{-1} \left\{ I + PK - PK \right\} r + \text{other stuff}$$

$$e_T = [I + PK]^{-1} \cdot r - [I + PK]^{-1} P d_i \\ - [I + PK]^{-1} d_o + [I + PK]^{-1} PK n$$

* $S \equiv [I + PK]^{-1} = \text{sensitivity TFM}$

$$e_T = S \left\{ r - P d_i - d_o + PK n \right\}$$

$r \rightarrow$ Always Low Frequency Commands !

Recall , we want :

- ① good low frequency command following
- ② good low frequency disturbance rejection
- ③ good high frequency sensor noise Attenuation

Note :

i) S small at low frequencies

- S must be small at freq where r has its energy content (low freq.)

- ii) S small where Pdi has its energy content.
S small where do has its energy content.
- iii) SPK small where -z has its energy content
(typically at high freq)

$$T \equiv [I + PK]^{-1} PK$$

$$= SPK$$

$$= I - S \quad \sim [I + PK]^{-1}$$

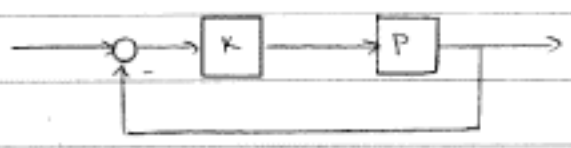
Note: $S + T = I$

SSISO check: $S = \frac{1}{1+PK} \quad T = \frac{PK}{1+PK}$

$\hookrightarrow S + T = 1$ ✓

T = complementary sensitivity matrix (TFM)

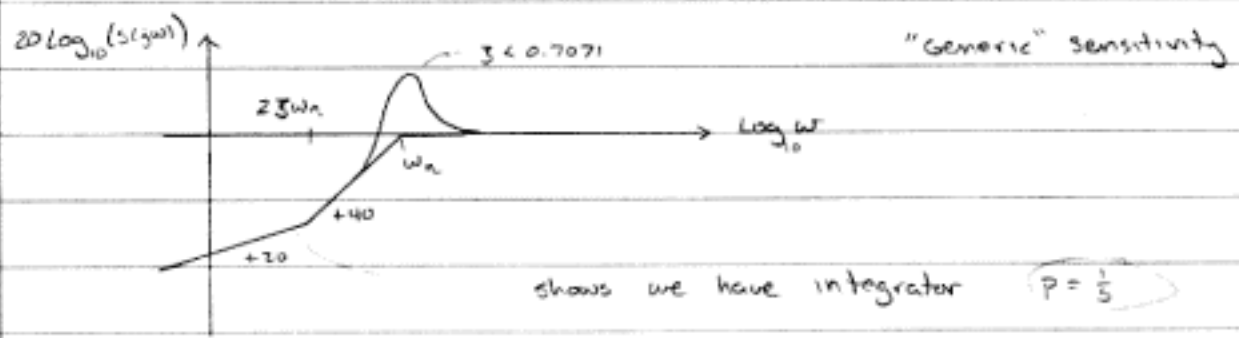
Example



$P = \frac{1}{s}$

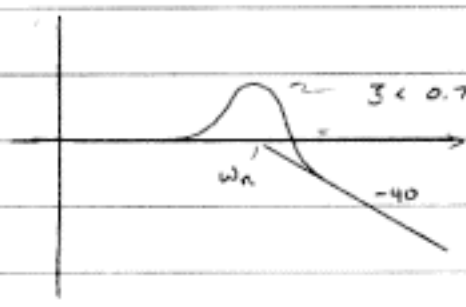
$K = \frac{\omega_n^2}{s + 2\zeta\omega_n}$

$S = \frac{1}{1+PK} = \frac{1}{1 + \frac{\omega_n^2}{s(s+2\zeta\omega_n)}} = \frac{s(s+2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ $\zeta < 0.7071$

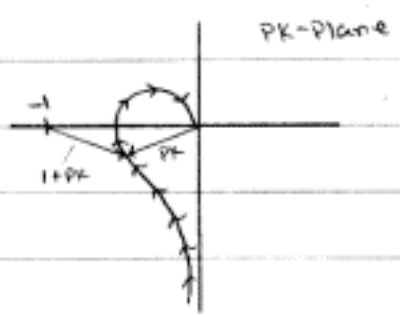
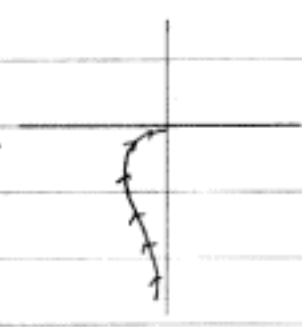
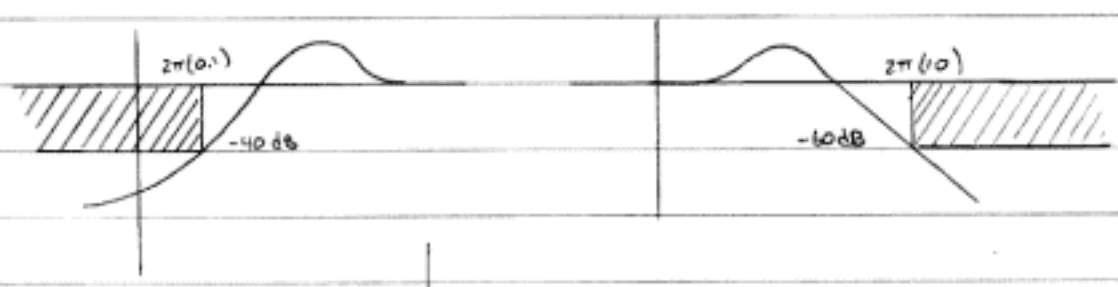


shows we have integrator $P = \frac{1}{s}$

$$T = \frac{PK}{1+PK} = 1-S = 1 - \frac{s(2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



"Generic"
complementary
sensitivity



PK-plane
dist from -1 to PK, $d(-1, PK)$
should be large!
(want $1+PK$ large)

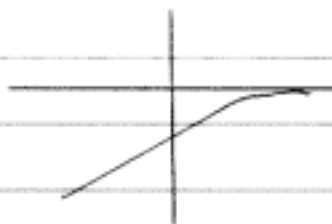
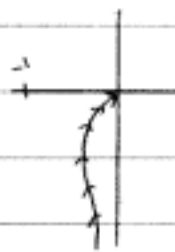
$$d(-1, PK) = |1+PK| = \frac{1}{|S|}$$

$$|S| = \left| \frac{1}{1+PK} \right| = \frac{1}{d(-1, PK)}$$

Robustness

$d(-1, PK)$ large ! \Leftrightarrow δ small

Robust



Not Robust

