EEE 598 — Tuesday 10 February 1998

Example

$$A = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1] \longleftarrow 10\ 1\text{'s}$$

Find $\bar{\sigma}[A]$ or $\|A\|_2$

Should we use $A^H A$ or AA^H ?

$$A^H A = \begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix}[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1] \Rightarrow [10 \times 10]\ !$$

$$A \cdot A^H = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix} = 10$$

$$\|A\|_2 = \bar{\sigma}[A] = \sqrt{10}$$

$$\boxed{\text{USE } AA^H \text{ in this case !}}$$

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A class of MIMO LTI systems

→ Introduction to state space

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

— state vector

— output vector

y = output vector $\in \mathbb{R}^p$

u = Input or Control vector $\in \mathbb{R}^m$

x = state vector $\in \mathbb{R}^n$

$A \cdot (n \times n)$ $\qquad C - (p \times n)$

$B - (n \times m)$ $\qquad D - (p \times m)$

Example #1

$$H(s) = \frac{1}{s+1} = \frac{Y(s)}{U(s)}$$

$$\dot{y} + y = u$$

let $x = y$

$$\dot{x} = -x + u$$

$$y = x$$

$A = -1 \qquad B = C = 1 \qquad D = 0$

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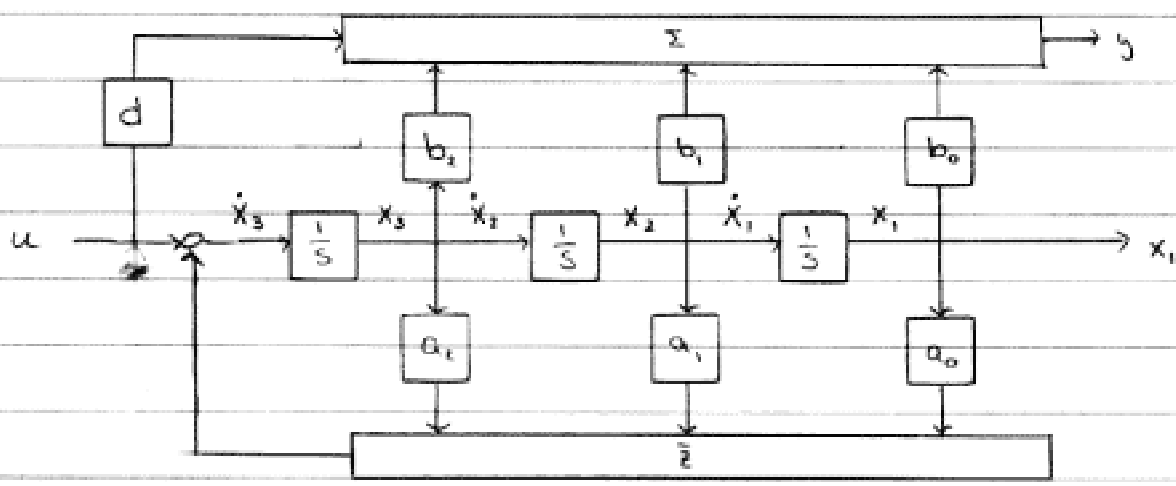
Example # 2

$$H(s) = d + \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 + a_0} = \frac{3^{rd} \text{ order num}}{3^{rd} \text{ order den}}$$

Determine a state space representation [A, B, C, D]

Solution :

$$H(s) = d + \frac{\frac{b_2}{s} + \frac{b_1}{s^2} + \frac{b_0}{s^3}}{1 + \frac{a_2}{s} + \frac{a_1}{s^2} + \frac{a_0}{s^3}}$$



$\dot{x}_1 = x_2$ $\qquad\qquad$ $\dot{x}_3 = -a_0 x_1 - a_1 x_2 - a_2 x_3 + u$

$\dot{x}_2 = x_3$

$y = b_0 x_1 + b_1 x_2 + b_2 x_3$

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Ex 2 cont.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_1 & -a_2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \qquad D = [d]$$

$\Rightarrow$ claim : $X(s) = (sI - A)^{-1} x_0 + (sI - A)^{-1} B\, U(s)$

$$Y(s) = C(sI - A)^{-1} x_0 + H(s)\, U(s)$$

$$H(s) = C(sI - A)^{-1} B + D$$