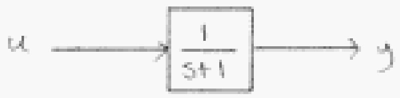
TRANSMISSION ZEROS

Definition: A system has a transmission zero at s_0 if there exists directions u_0 & x_0 s.t.

input dir. state dir.

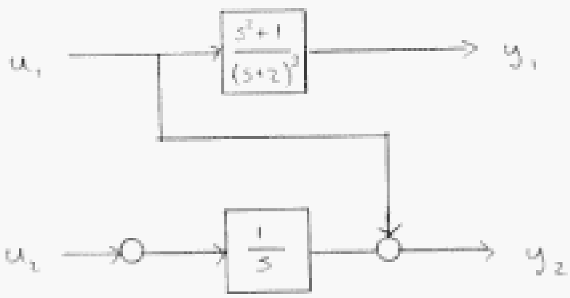
(not both zero) if $u(t) = u_0 e^{s_0 t}$
then $x(t) = x_0 e^{s_0 t}$ & $y(t) = 0$ for all $t > 0$.

SISO



Input is a Scalar
Direction is not an issue

MIMO



here, direction of $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ is an issue

If $u_1 = \sin t \Rightarrow$ get $y_1 = 0$
$(y_2(t) \neq 0)$

- Transmission zeros for MIMO LTI Systems

Consider the MIMO LTI system described by (A,B,C,D)

CLAIM: The above system has a transmission zero at s_0 iff the matrix does not have full rank (rank < min (n+p, m+n))

$$\begin{array}{c} n \updownarrow \\ p \updownarrow \end{array} \begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} \quad [(n+p) \times (n+m)]$$

$$\qquad \overset{\leftrightarrow}{n} \qquad \overset{\leftrightarrow}{m}$$

Note: We will focus almost exclusively on the case where $m = p$

[# of inputs = # of outputs]

ie, "square systems"

For such systems,

s_0 is a TZ iff $\det \begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} = 0$

The "zero direction" $\begin{bmatrix} x_0 \\ u_0 \end{bmatrix}$ can be found by solving

$$\begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} \begin{bmatrix} x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Example

$$H(s) = \frac{s+1}{s} = 1 + \frac{1}{s} \qquad (\text{zero @ } s = -1)$$

$$A = 0 \qquad B = 1 \qquad C = 1 \qquad D = 1$$

check,

$$C(sI-A)^{-1}B + D = 1(sI-0)^{-1}1 + 1 = \frac{1}{s} + 1 \quad \checkmark$$

- Let's determine the TZ's of this (A, B, C, D) LTI system.

$$\det \begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} = \det \begin{bmatrix} s_0 - 0 & -1 \\ 1 & 1 \end{bmatrix} = s+1 = 0$$

$$\boxed{TZ \text{ @ } s_0 = -1} \qquad \text{As we expected !}$$

- Let's compute the so-called "zero-direction" $\begin{bmatrix} x_0 \\ u_0 \end{bmatrix}$

$$\begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} \begin{bmatrix} x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

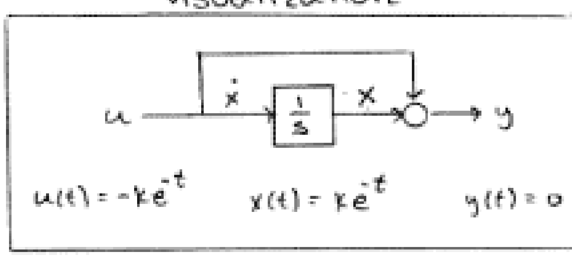
$$\begin{bmatrix} x_0 \\ u_0 \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \text{"general solution"} \quad k = \text{constant}$$

Interpretation :

system :

$$\dot{x} = 0x + 1u$$
$$y = 1x + 1u$$

visualization



If $u(t) = -ke^{-1t}$

then,

$$x(t) = ke^{-1t} \quad \& \quad y(t) = 0 \quad \text{for all } t > 0$$

Question

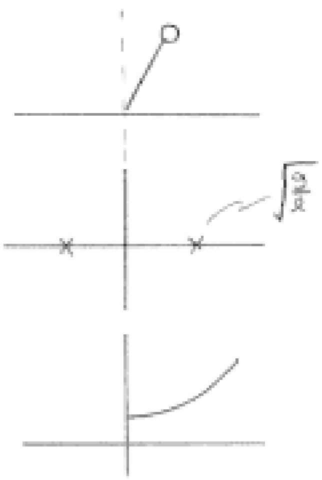
For SISO systems, we can read T.Z.'s from the T.F.

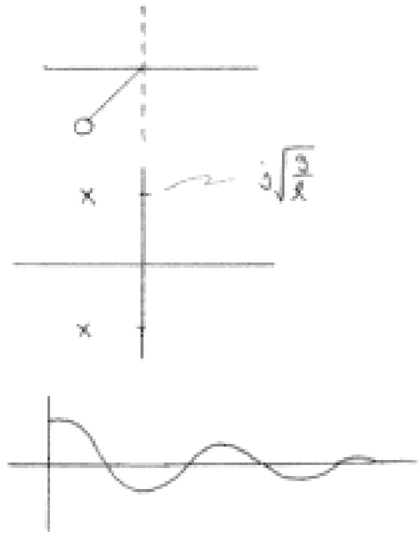
Can we read (in general) the T.Z.'s from the entries of a MIMO TFM?

$\boxed{\text{NO !}}$

POLES : Natural Modes

All physical systems
- natural modes
- natural tendencies
- poles





We need to quantify the above ideas

$\dot{x} = Ax \qquad x(0^-) = x_0$

Fact $\qquad x(t) = \underbrace{e^{At}}_{\text{matrix exponential}} x_0$