

Second Order Systems

Consider the general second order system:

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where:

ζ (zeta) - damping ratio
 ω_n - Undamped natural frequency

- 1) i) Plot the poles of the general second order transfer function for $\zeta=0.7071$ and $\omega_n=0:0.25:3$



Have this plot initialized by the T.A.

- ii) Plot the poles of the general second order transfer function for $\zeta=0.5$ and $\omega_n=0:0.25:3$



Have this plot initialized by the T.A.

Question: What shape in the s-plane does a plot of the poles form for a constant ζ ?

Question: What is the angle between the plot of the poles and the negative x-axis for part (i)? Part (ii)?

- 2) i) Plot the poles of the general second order transfer function for $\omega_n = 2$ and $\zeta= 0:0.1:2$



Have this plot initialized by the T.A.

- ii) Plot the poles of the general second order transfer function for $\omega_n = 3$ and $\zeta= 0:0.1:2$



Have this plot initialized by the T.A.

Question: What shape in the s-plane does the plot of the poles form for constant ω_n ?

Question: What happens to the poles as ζ continues to increase?

- 3) Step responses for the general second order system

- i) Create a graph of the second order system step response for $\zeta=0.7071$ and $\omega_n=0:0.25:4$ for $t=0:0.01:5$



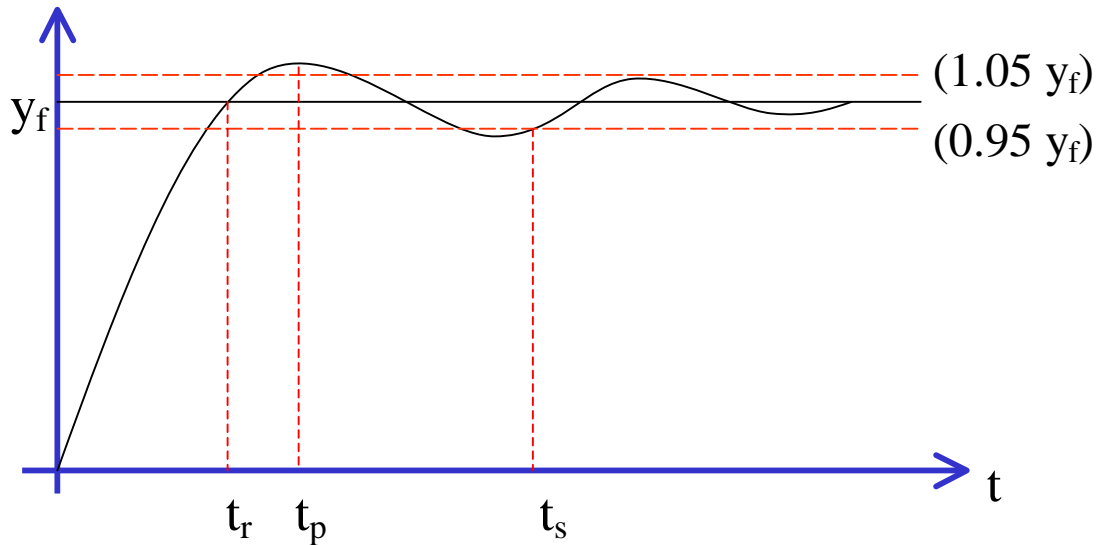
Have this plot initialized by the T.A.

- ii) Create a graph of the second order system step response for $\omega_n=2$ and $\zeta=0:0.25:3$ for $t=0:0.01:5$



Have this plot initialized by the T.A.

- 4) Describing a transient response



Rise Time: Measures the quickness of the response of the system. There are several different rise-times (e.g. time for a system to go from 10% to 90% of the final value, time for response to go from 0% to 100% of the final value, etc.)

Peak time: Measure of the quickness of the system response. The time between the application of the step input and the output attaining its peak value.

Settling time: Time after which the output remains within a fixed bound of the final value. A typical measure is the 5% settling time.

Create a graph of the step response for the following system:

$$\frac{2600}{4s^4 + 128s^3 + 1053s^2 + 1990s + 2600}$$

Print this graph and label the final value (y_f), peak value (y_p), rise time (t_r), peak time (t_p), and the 5% settling time (t_s). Please include the $\pm 5\%$ envelope of the final value on the graph.



Have this plot initialized by the T.A.

Question: What are the percent overshoot, rise-time, peak time, and 5% settling time?

5) Second order general systems

Using the step response of the second order system expressions for the above transient response, descriptors can be developed. Let's look at these relationships.

- i) Create a graph of peak time, t_p , for $\omega_n=1$ and $\zeta=0:0.001:0.999$.
Hint:

$$t_p = \frac{\pi}{\sqrt{1-\zeta^2}}$$



Have this plot initialized by the T.A.

- ii) Create a graph of the percent overshoot, M_p , for the second order system for $\omega_n=1$ and $\zeta=0:0.01:0.999$
Hint:

$$M_p = (100) \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

- iii) Create a graph of the 2% settling time for the general second order system with $\omega_n=1$ and $\zeta=0:0.01:2$
Hint:

$$t_s = \frac{4}{\zeta\omega_n}$$

Question:

Which ζ values have the fastest peak times?

How do the locations of the poles corresponding to the fast peak times relate to the locations of the poles corresponding to the slow peak times?

Question:

Which ζ values have the largest overshoot?

Do these ζ values correspond to the systems with fast or slow peak times?

Question:

Which ζ values have the longer settling times?

Do these ζ values correspond to the systems with fast or slow peak times?

6) Second order system with single zero

For the system transfer function:

$$\frac{Y(s)}{R(s)} = \frac{\frac{1}{a}s + 1}{s^2 + s + 1}$$

- i) Create a graph of the step response of y as a varies.
Do this for $a = \{0.3, 0.4, 0.5, 1.0, 2.0, 10^{10}\}$



Have this plot initialized by the T.A.

- ii) Create a bode plot for this system as a varies.
Do this for $a = \{0.3, 0.4, 0.5, 1.0, 2.0, 10^{10}\}$



Have this plot initialized by the T.A.

- iii) Create a pole-zero plot for this system as a varies.
Do this for $a = \{0.3, 0.4, 0.5, 1.0, 2.0, 10^{10}\}$
- Plot the zeros in the s -plane as green 'o's
Hint: `plot(real(z), imag(z), 'go')`
 - Plot the poles in the s -plane as red 'x's
Hint: `plot(real(z), imag(z), 'rx')`
 - Label the poles and zeros using the 'text' or 'gtext' command
 - Overlay the multiple plots on a single graph.



Have this plot initialized by the T.A.

Question: Describe how the step responses and frequency responses are affected by the position of the zero in the s -plane.