Lab 5 - Root Locus II

March 31, 1998

1 Introduction

The purpose of this lab is to introduce MATLAB commands to help analyze more complicated root loci.

New MATLAB commands:

- $[\mathbf{k}, \mathbf{poles}] = \mathbf{rlocfind}(\mathbf{num}, \mathbf{den})$ Places a crosshair cursor in the graphics window which is used to select a pole location on the existing root locus. The root locus gain associated with the selected point is returned in **k** and all the system poles for this gain are returned in **Poles**.

- $\mathbf{C} = \mathbf{conv}(\mathbf{A}, \mathbf{B})$ Convolves vectors A and B. If A and B are polynomial coefficients, convolving them is equivalent to multiplying the two polynomials.

- $[\mathbf{Q}, \mathbf{R}] = \mathbf{deconv}(\mathbf{B}, \mathbf{A})$ Deconvolves vector A out of vector B. The result is returned in a quotient vector Q and remainder vector R. If A and B represent polynoimial coefficents, deconvolution is equivalent to polynomial division. The result of dividing B by A is the quotient polynomial Q and remainder polynomial R.

- $[\mathbf{numc}, \mathbf{denc}] = \mathbf{cloop}(\mathbf{numkp}, \mathbf{denkp}, \mathbf{sign})$ Produces the SISO closed loop system in transfer function form obtained with unity feedback with the sign sign. If sign = -1 then negative feedback is used. Use **cloop** if you are using MATLAB version 4.2c.

- $[\mathbf{numc}, \mathbf{denc}] = \mathbf{feedback}(\mathbf{numpk}, \mathbf{denpk}, \mathbf{num2}, \mathbf{den2}, \mathbf{sign})$ Produces the SISO closed loop system in transfer function form obtained by connecting the two SISO transfer function systems in feedback with the sign **sign**. If **sign** = -1 then negative feedback is used. This command is equivalent to **cloop** for **num2** = 1 and **den2** = 1. Use **feedback** if you are using MATLAB version 5.

2 Plant $p_1(s)$

Consider a system with the following plant

$$p_1(s) = \frac{\frac{2}{\alpha}(\alpha - s)}{s(s+2)} \tag{1}$$

where α = (20, 0.1, -1, -2.1, -20)

- Create a separate root locus plot for the above system with each α. Label each graph with the plant transfer function and *alpha* value. Label your axes (real and imaginary).

  Get this graph initialed by the TA and turn in as part of your lab report.

  Question Sketch the complimentary root locus for these system corresponding to each α?

  Question How would you use Matlab to create the complementary root locus plot?

- Use the MATLAB command *rlocfind* to find the breakpoints, imaginary axis, and range of k correpsonding to stable systems with α = (20, 0.1, -1, -2.1, -20). The report must present this data in a table. The table must have columns for α, break points, imaginary crossovers, stable range of k.

- Analytically find the break points, imaginary crossovers and the range of k correpsonding to stable systems. Only consider the above system for α = (20, -2.1).

  Note: This step can be done outside the lab but must be included with the lab report. The report must present this data in a table. The table must have columns for α, break points, imaginary crossovers, and range of k for a stable system.

  Question Compare the break points and imaginary crossovers found analytically and found using MATLAB. Are there differences? Why?

- Find the output step response for the closed loop system for the following system parameters $(\alpha = 20, k = 25)$, $(\alpha = -2.1, k = 1)$, $(\alpha = -2.1, k = 2)$, and $(\alpha = -2.1, k = 5)$. Use MATLAB to calculate the closed loop transfer function,

  $$\frac{PK}{1+PK}. \tag{2}$$

  The *cloop* or *feedback* commands evaluate the numerator and denominator of this transfer function given the open loop transfer function, PK. Use the *cloop* or *feedback* output numerator and denominator with the *step* command to find the output step response of the closed loop system. Create two graphs, one for the $(\alpha = 20, k = 25)$ case and one for the $(\alpha = -2.1, k = 1, 2, 5)$ cases.

  It is important to title these graphs, label the axes, and label the each plot (with appropriate k values) using MATLAB.

  Hint: You can adapt the following MATLAB code for use in your script file to label the plots.

  ```
  rpmstring = ['K = ' num2str(k)];
  gtext(rpmstring)
  ```

  Get this graph initialed by the TA and turn in as part of your lab report.

3 Plant $p_2(s)$

Consider a system with the following plant

$$p_2(s) = \frac{5}{s(s+2)(s^2+2s+5)} \tag{3}$$

- Use MATLAB's *conv* command to multiply the denominator terms.
- Create a root locus plot for this system, $p_2(s)$.
- Find the closed loop poles, roots of $1 + PK$, for $k = (1, 1.5, 2, 2.5, 5)$. Plot these poles on the root locus plot created in the previous step. Label the roots with the correpsonding k value.

  Get this graph initialed by the TA and turn in as part of your lab report.

  At home create a table of the k values and correpsonding roots. Turn this in as part of the report.

- Using *rlocfind* find the break points, imaginary crossovers, range of k corresponding to a stable system.
- Using *step* with *cloop* or *feedback* find the output step response of this system for $k = (1, 1.5, 2, 2.5, 5)$. Plot the step responses on one graph. It is important to title this graph, label the axes, and label the each step response (with appropriate k values) using MATLAB.

  Get this graph initialed by the TA and turn in as part of your lab report.

4 Plant $p_3(s)$

Consider a system with the following plant

$$p_3(s) = \frac{2s(s+1)}{s(s-1)(s^2+4s+16)} \tag{4}$$

- Use MATLAB's *conv* command to multiply the denominator terms and to multiply the numerator terms.
- Create a root locus plot for this system, $p_3(s)$.
- Find the closed loop poles, roots of $1 + PK$, for $k = (0.1, 1.2, 2.5, 3, 5)$. Plot these poles on the root locus plot created in the previous step. Label the roots with the correpsonding k value.

  Get this graph initialed by the TA and turn in as part of your lab report.

  At home create a table of the k values and correpsonding roots. Turn this in as part of the report.

- Using *rlocfind* find the break points, imaginary crossovers, range of k corresponding to a stable system.

- Using *step* with *cloop* or *feedback* find the output step response of this system for k = (0.1, 1.2, 2.5, 3, 5). Plot the step responses on one graph. It is important to title this graph, label the axes, and label the each step response (with appropriate k values) using MATLAB.

  Get this graph initialed by the TA and turn in as part of your lab report.

5 Plant $p_4(s)$

Consider a system with the following plant

$$p_4(s) = \frac{(s+5)(s^2+20s+325)}{(s-1)(s-3)(s^2+8s+416)(s^2+30s+450)} \quad (5)$$

- Use MATLAB's *conv* command to multiply the denominator terms and to multiply the numerator terms.

- Create a root locus plot for this system, $p_4(s)$.

- Find the closed loop poles, roots of $1 + PK$, for k = (1000, 2000, 3500, 6000, 8000). Plot these poles on the root locus plot created in the previous step. Label the roots with the correpsonding k value.

  Get this graph initialed by the TA and turn in as part of your lab report.

  At home create a table of the k values and correpsonding roots. Turn this in as part of the report.

- Using *rlocfind* find the break points, imaginary crossovers, range of k corresponding to a stable system.

- Using *step* with *cloop* or *feedback* find the output step response of this system for k = (1000, 2000, 3500, 6000). Plot the step responses on one graph. It is important to title this graph, label the axes, and label the each step response (with appropriate k values) using MATLAB.

  Get this graph initialed by the TA and turn in as part of your lab report.