

EEE 480: Feedback Systems
Laboratory Assignment # 8

Design and Analysis of a Pitch Attitude Control System for a Sikorsky UH-60A Blackhawk Helicopter

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1 Introduction

In this laboratory assignment, we will design and analyze a pitch-attitude control system for a Sikorsky UH-60A Blackhawk Combat Assault Helicopter. Please answer all of the questions in each section, and include all of your plots in your report. MATLAB version 5.0 (or higher) will be used in this lab.

It should be noted that the questions which are asked below are questions which any control system engineer should ask - regardless of the physical system which they are considering. In this sense, they are standard questions.

2 Sikorsky UH-60A Blackhawk Longitudinal Dynamics

In this section, you will analyze the longitudinal (fore-aft-pitching) dynamics for a Sikorsky UH-60A Blackhawk. For simplicity, we consider a simplified linear model which is valid near hover. The helicopter input (i.e. control) is called the *cyclic pitch control* and is denoted by the symbol B_{lc} . This control is used to control the effective “tilt” of the helicopter’s main rotor. This indirectly controls pitching moments about the helicopter’s center of gravity, the pitch attitude, and the helicopter’s speed.

The transfer function from the cyclic control ($u = B_{lc}$, measured in *radians*) to the pitch-attitude (θ , measured in *radians*) is:

$$P(s) = \frac{\theta}{B_{lc}} = \frac{-47.24s - 1.711}{s^3 + 3.16s^2 + 0.186s + 1.324}. \quad (1)$$

This transfer function will be referred to as the *plant transfer function*. It will be used as the basis for your pitch attitude control system design.

The transfer function from the control $u = B_{lc}$ (measured in radians) to the speed \dot{x} (measured in *ft/sec*) is given by:

$$\frac{\dot{x}}{B_{lc}} = \frac{27.4s^2 + 84.9s + 1525}{s^3 + 3.16s^2 + 0.186s + 1.324}. \quad (2)$$

3 Pitch Attitude Control System Design and Analysis

In this section, you will design a pitch attitude control system for the Blackhawk. We want our closed-loop system to meet the following specifications:

- zero steady state error to step commands
- no, or minimal overshoot in the output (pitch attitude, θ)
- 5% rise-time of approximately 2.5 seconds

Controller design is often an iterative process. First, a nominal design is found, and then it is modified in order to meet the design specifications. Coming up with a good nominal design can take time and experience with the system being designed for. However, a good understanding of the basic analytical tools and concepts can help give more insight into the problem. In this lab, we will proceed through a systematic way for designing our pitch-attitude control system for the Blackhawk helicopter system.

The first issue in designing the controller is to pick a structure for it. This control system design will be based on the block diagram shown in Figure 1. The series compensator, K , and the command pre-filter, W , are assumed to possess the following structure:

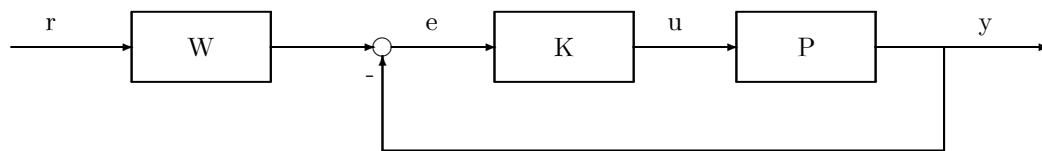


Figure 1: Visualization of Pitch Attitude Control System To Be Designed

$$K(s) = k \left[\frac{(s+a)^2}{s} \right] \left[\frac{50}{s+50} \right]^3 \quad (3)$$

$$W(s) = \left[\frac{a}{s+a} \right]^2 \quad (4)$$

Let us examine the preliminary design. If we set $a = 1$ and $k = -1$, and calculate the open-loop system:

$$L(s) = P(s)K(s). \quad (5)$$

we can plot the root-locus associated with this system. It is shown in Figure 2.

An analysis of the closed-loop transfer function will show that it has two dominant (conjugate-pair) poles, as shown in Figure 2. Thus, the closed-loop system may be approximated by a second-order system. The goal is to design our controller such that the *dominant* closed-loop poles are at the locations corresponding to our design specifications. Second-order systems are great, because we can relate our design specifications to the pole-locations. Let's do this.

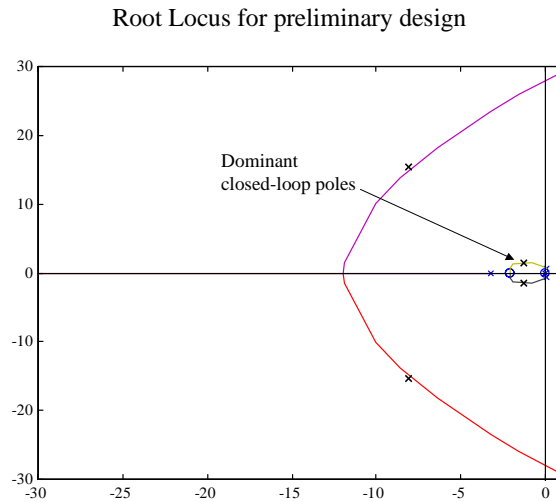


Figure 2: Preliminary Root-Locus

1. We will set the following specifications for our nominal design:

- 5% rise-time of $t_r = 2.5$ seconds
- overshoot, $M_p = 0.2\%$

Calculate the ω_n and ζ that will meet these specifications for a second-order system. What are the target closed-loop poles?

2. Now that we know where we want to place our dominant closed-loop poles, we can design our controller to do just this. We will take an empirical approach to the design. Essentially, we have two parameters to select: a and k . Let's see how we can select these parameters to place the closed-loop poles on their target locations.

For $k = 1$, plot the root-locus for the system for $a = 1.8, 1.9, 2.0, 2.1, 2.2$ overlaid on the same plot. Change the axis to just focus on the small 'loop' on the plot. Hint: Sse the MATLAB command `axis([-3 3 -2 2]);` to do this.

How does a affect the 'trajectory' (or locus) of the closed-loop poles?

From above, we know that our target closed-loop poles are approximately at $-2 \pm j1$. Select a and k such that the closed-loop poles are at these locations. Hint: Select a (up to two decimal places will do) such that the locus passes through the target point, then use the `rlocfind` command to find the k that will put the closed-loop poles at that point.

3. What are the poles and zeros of your designed compensator K ?
4. Plot the frequency response of your compensator, K .
5. Use MATLAB to construct a root locus for your design. Hint: Use the `rlocus` command in MATLAB. Use classical root locus rules to discuss the generated plot.
6. What are the imaginary crossovers on the root locus? Hint: Use the `rlocfind` command in MATLAB.
7. Use MATLAB to plot the open loop frequency response.

8. What is the phase crossover frequency ω_{p_1} associated with your design; i.e. at what frequency ω_{p_1} is

$$\angle L(j\omega_{p_1}) = -180 \pm 360 \text{ deg?} \quad (6)$$

9. Use MATLAB to compute $k_1 = \frac{1}{|L(j\omega_{p_1})|}$. What is the significance of k_1 and ω_{p_1} on the root locus plotted above?
10. What is the *unity gain crossover frequency* associated with your design; i.e. the lowest frequency, ω_g , such that

$$|L(j\omega_g)| = 1(0\text{db})? \quad (7)$$

11. What is the *phase margin PM* associated with your control system design? Recall that

$$PM \stackrel{\text{def}}{=} 180 + \angle L(j\omega_g) \text{ deg.} \quad (8)$$

What is the significance of the phase margin?

12. What is the downward gain margin, $\downarrow GM$? What is its physical significance?
13. What is the upward gain margin, $\uparrow GM$? What is its physical significance?
14. What information does the *margin* command in MATLAB provide? How do the results of the *margin* command relate to what you have computed above?
15. Plot the sensitivity frequency response. Recall that

$$S \stackrel{\text{def}}{=} \frac{1}{1 + PK} \quad (9)$$

16. Over what frequency range is $|S(j\omega)| \leq 0.1$ (-20 db)?
17. What is the transfer function from r to the tracking error,

$$e_t \stackrel{\text{def}}{=} r - y? \quad (10)$$

Please note that it is not S .

18. Why doesn't the sensitivity directly tell us about low frequency command following? Hint: Do not forget the definition of the tracking error and the effect of the pre-filter W .
19. What is the approximate steady state output when the pitch attitude reference command is $r(t) = \sin 2t$? Hint: You can read this right off of the bode plot.
20. Plot the frequency response for the transfer function from r to the tracking error, e_t (computed above).
21. Suppose that a reference pitch attitude command $r(t) = A \sin(\omega t + \theta)$ is issued to the control system. For what ω values will the steady state tracking error $e_{t_{ss}}$ satisfy: $|e_{t_{ss}}| < 0.1 A$?
22. What is the approximate steady state tracking error, $e_{t_{ss}}$, when a reference command $r(t) = \sin 2t$ is issued to the control system. Is your control system design intended to operate with a reference command r of this frequency? Explain.
23. What is the transfer function from r to the control, u ? Plot the frequency response for this transfer function. Discuss how reference commands are attenuated in producing the control.

24. Plot the complementary sensitivity frequency response. Recall that

$$T(s) \stackrel{\text{def}}{=} 1 - S(s). \quad (11)$$

25. Use MATLAB to form the transfer function from r to y . Plot the frequency response associated with this transfer function. What are the poles and zeros of the transfer-function from r to y ? If you cancel-out the poles with the zeros that are close to them, what are you left with? Does this ‘reduced’ system correspond with our assumption of a pair of dominant poles? Compute our ‘target’ transfer function:

$$L_{\text{target}}(s) = \frac{5}{(s + 2 - j)(s + 2 + j)} \quad (12)$$

and plot its frequency response over the frequency response of our closed-loop system above. Do they match closely?

26. Plot the output response to a step command of 10 *degrees*. What is the rise-time and over-shoot for this system? How closely do they match our design specifications? Note: Our simplifying assumptions have taken us this far, at this point, additional analysis needs to be done if the performance needs to be improved upon further.