

# Multivariable Analysis and Control of a Cart-Pendulum-Seesaw System Using an Animation Tool

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## Abstract

This paper describes a software environment for simulating, visualizing, and evaluating robust multivariable controller performance for a cart-pendulum-seesaw system. Different simulation and animation models may be selected by the user. Users are also able to alter model and controller parameters on the fly - thus allowing them to quickly examine different scenarios. Microsoft Direct3D three dimensional graphics are used for animation. Graphical indicators and real-time plotting are also employed to help users abstract-out key phenomena. The environment also accommodates data exchange with MATLAB. The *interactive MoSART Cart-Pendulum-Seesaw (CPS) Environment* is shown to be a valuable tool for enhancing both research and education. Examples are presented to illustrate its utility.

## 1 Introduction: The Need for Interactive Visualization Tools

Today's affordable PC technology, object oriented programming languages, and other software development tools now permit the development of new interactive graphical visualization environments - environments which could revolutionize systems and controls analysis, design, research, and education. These technologies have been exploited by MoSART

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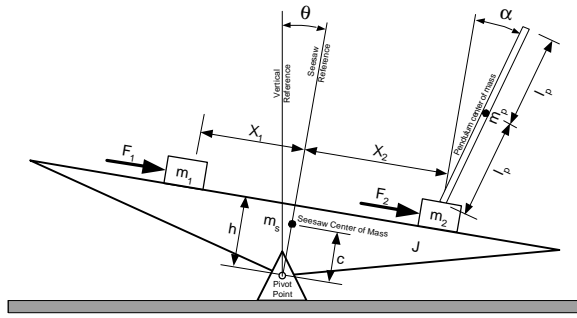
researchers at ASU to develop several system-specific environments for a variety of systems [4]-[6],[8]-[10],[13],[14]. This paper describes the development of an *Interactive MoSART Cart-Pendulum-Seesaw Environment*.

**The Need For Design Tools.** Because of increased performance requirements, the need for systematic multivariable control system design techniques [16] which suitably accommodate multiple-input multiple-output (MIMO) cross-coupling effects and uncertainty have become essential.

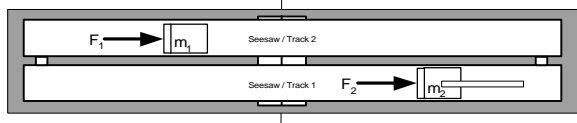
Since control system design is a highly iterative process, tools which facilitate the evaluation and visualization of MIMO control laws would be invaluable to designers. Given this, tools which simultaneously accommodate interactive modeling, simulation, analysis, graphical visualization, animation, design, and real-time control features are of particular use to designers. Only until very recently, the development of such tools has been restricted to mainframe and workstation platforms.

**Contributions of Work.** This paper demonstrates how affordable state-of-the-art PC technologies may be combined to develop high quality system-specific *Interactive MoSART Environments* which are useful for enhancing the research and education process. Specifically, this paper describes a PC/Windows NT/95/Visual C++/MATLAB/SIMULINK-based *Interactive MoSART Cart-Pendulum-Seesaw (CPS) Environment*. This environment permits control system engineers and students to analyze, design, and visualize the performance of controllers for complex mechanical systems via real-time and faster-than-real-time animation.

**Outline.** The remainder of this paper is organized as follows. Section 2 describes the mathematical model and control laws that have been implemented within the *Interactive MoSART Cart-Pendulum-Seesaw (CPS) Environment*. In section 3, the environment's key features are described. In section 4, the utility of the environment as a research and educational tool is demon-



(a) Side View



(b) Top View

Figure 1: Visualization of Cart-Pendulum-Seesaw System

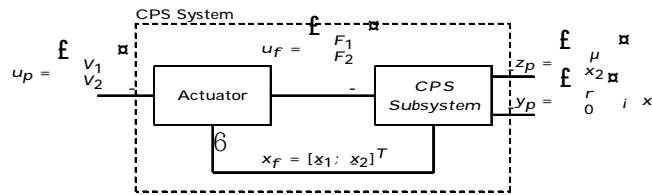


Figure 2: Block Diagram Visualization of CPS-System

strated. Section 5 summarizes the paper and presents directions for future research.

## 2 The CPS Model and Control Laws

This section describes mathematical models and control laws which are implemented within our *Interactive MoSART Cart-Pendulum-Seesaw (CPS) Environment*. Mathematical models include a nonlinear model (including actuator dynamics) as well as a corresponding linearized model. A linear MIMO  $H^1$  controller is implemented within the environment. The  $H^1$  design methodology weighting functions may be altered within the environment to redesign the controller (using MATLAB's robust control toolbox [2]).

**Description of System.** The system to be considered in this work may be visualized as shown in Figure 1. The system consists of two mobile carts which are coupled via rack and pinion mechanisms to two parallel tracks mounted on parallel seesaws. The seesaws are joined and are free to rotate in unison about the pivot point indicated in Figure 1. One cart carries an inverted

pendulum which is free to fall. The other cart serves as a counterbalance. Forces are applied to each cart via DC motors. Each motor is controlled with a voltage. The goal is to use these voltages to independently control the seesaw angle ( $\mu$ ) and the pendulum-carrying cart position ( $x_2$ ) while keeping the pendulum upright. The natural instabilities and MIMO nonlinear coupling associated with this system make this task impossible for a human being. This provides motivation for an automatic control system.

### 2.1 CPS Nonlinear Model

Lagrangian methods [3] were used to derive an eight order nonlinear dynamical model for the CPS-System. This model may be visualized as shown in Figure 2 and may be written as follows [12]:

$$\dot{x} = F(x; u_f) \quad u_f = K_{vf} u_p - K_{be} x \quad z_p = Cx \quad (1)$$

where the voltage control vector  $u_p \in \mathbb{R}^{2 \times 1}$ , the resulting force vector  $u_f \in \mathbb{R}^{2 \times 1}$ , the state vector  $x \in \mathbb{R}^{8 \times 1}$ , the internal feedback vector  $x_f \in \mathbb{R}^{2 \times 1}$ , the output vector  $z_p \in \mathbb{R}^{2 \times 1}$ , the reference command vector  $r \in \mathbb{R}^{2 \times 1}$  and the feedback vector  $y_p \in \mathbb{R}^{8 \times 1}$  are defined as follows:

$$u_p = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \begin{matrix} i \\ i \end{matrix} \quad \begin{matrix} \text{voltage applied to motor 1 (volts)} \\ \text{voltage applied to motor 2 (volts)} \end{matrix} \quad (2)$$

$$u_f = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad \begin{matrix} i \\ i \end{matrix} \quad \begin{matrix} \text{force over cart 1 (new tons)} \\ \text{force over cart 2 (new tons)} \end{matrix} \quad (3)$$

$$x = \begin{bmatrix} \mu \\ x_2 \\ \dot{\mu} \\ x_1 \\ \dot{x}_2 \\ \dot{\mu} \\ \dot{x}_1 \end{bmatrix} \quad \begin{matrix} i \\ i \\ i \\ i \\ i \\ i \\ i \end{matrix} \quad \begin{matrix} \text{seesaw angle (rad=sec)} \\ \text{cart 2 position (m)} \\ \text{pendulum angle (rad)} \\ \text{cart 1 position (m)} \\ \text{seesaw angle rate (rad=sec)} \\ \text{cart 2 velocity (m=sec)} \\ \text{pendulum angle rate (rad=sec)} \\ \text{cart 1 velocity (m=sec)} \end{matrix} \quad (4)$$

$$x_f = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \quad \begin{matrix} i \\ i \end{matrix} \quad \begin{matrix} \text{cart 1 velocity (m=sec)} \\ \text{cart 2 velocity (m=sec)} \end{matrix} \quad (5)$$

$$z_p = \begin{bmatrix} \mu \\ x_2 \end{bmatrix} \quad \begin{matrix} i \\ i \end{matrix} \quad \begin{matrix} \text{seesaw angle (rad)} \\ \text{cart 2 position (m)} \end{matrix} \quad (6)$$

$$r = \begin{bmatrix} \mu_c \\ x_{2c} \end{bmatrix} \quad \begin{matrix} i \\ i \end{matrix} \quad \begin{matrix} \text{commanded seesaw angle (rad)} \\ \text{commanded cart 2 position (m)} \end{matrix} \quad (7)$$

$$y_p = \begin{bmatrix} r \\ 0_{6 \times 1} \end{bmatrix} \quad \begin{matrix} i \\ i \end{matrix} \quad \begin{matrix} \text{reference command vector} \\ \text{feedback vector} \end{matrix} \quad (8)$$

In the above model, the internal feedback  $x_f$  is a consequence of back electromotive forces due to inductive windings within the motors which generate the forces  $F_1$  and  $F_2$ . It should be noted that the above model does not include motor dynamics (states); i.e. each motor was modeled by a static map depending on the motor input voltages  $V_1, V_2$ , the cart velocities  $\dot{x}_1, \dot{x}_2$  and the actuator parameters  $K_{vf}$  and  $K_{be}$ . The output vector  $z_p = [\mu \ x_2]^T$  contains the variables which we are interested in controlling. The feedback vector  $y_p$  contains quantities which are assumed to be available to the control system.

Pole Description	System (w/ith actuator)	Subsystem (w/ /o actuator)
Pendulum Toppling	$s_1 = 6:0369$	$s_1 = 6:9121$
Seesaw Toppling	$s_2 = 3:0651$	$s_2 = 3:6615$
Damped-Seesaw Oscillation	$s_{3,4} = j 1:6859 \pm 51:5444i$	$s_{3,4} = \pm 3:4932i$
Cart-Position Equilibrium	$s_5 = 0:0000$	$s_5 = 0:0000$
Cart-Speed Damping	$s_6 = j 9:4340$	$s_6 = 0:0000$
Pendulum Damping	$s_7 = j 17:8801$	$s_7 = j 6:9121$
Seesaw Damping	$s_8 = j 5:0763$	$s_8 = j 3:6615$

Table 1: CPS-System (and Subsystem) Poles

### 2.2 CPS Linear Model

Linearizing the CPS-System about the equilibrium point  $x = 0$  and  $u = 0$  (i.e. all states and voltages equal to zero), results in the following linear state space representation:

$$\dot{x} = Ax + Bu_p \quad z_p = Cx \quad (9)$$

where  $A \in \mathbb{R}^{8 \times 8}$ ,  $B \in \mathbb{R}^{8 \times 2}$ , and  $C \in \mathbb{R}^{2 \times 8}$ , and all variables represent small perturbations with respect to equilibrium values.

**Linear Model Analysis.** The poles of the CPS-System are given in Table 1. In order to assist us in our discussion, we have included the CPS-Subsystem poles.

**Poles.** Associated with the CPS system are two (2) right half plane poles. These poles correspond to the natural "toppling tendencies" of the pendulum and seesaw. The pendulum's natural toppling tendency is greater and hence its associated pole is deeper in the right half plane. Two complex conjugate poles correspond to the seesaw's natural tendency to oscillate. Associated with the carts we have a position equilibrium (integrator) pole and a speed damping pole. These poles (or modes) are characteristic of any body which is undergoing translational motion through a viscous medium. The pendulum and seesaw each possess (angular velocity) damping poles. These poles (or modes) are characteristic of any body which is undergoing rotational motion through a viscous medium.

To fully understand the physical significance of the CPS system poles (or modes), it is instructive to consider the unactuated system; i.e. the CPS-Subsystem. One sees that the effect of the actuator system (i.e. motors) is to introduce damping into the system. The significant

impact which the actuator and specifically, the internal feedback gain  $k_{be}$ , has on the speed damping, pendulum damping, and seesaw oscillation poles is clearly seen in Table 1. Without the actuator, each of these poles lie further to the right. The speed damping pole becomes a pole at the origin and is referred to as a speed equilibrium mode. The seesaw damped oscillation poles move onto the imaginary axis yielding an undamped seesaw oscillation.

**Transmission Zeros.** The transmission zeros of the CPS-System ( $u_p$  to  $z_p$ ) are identical to those of the CPS-Subsystem ( $u_f$  to  $z_p$ ). The system has two (2) right half plane zeros and two (2) left half plane zeros. The set is symmetric with respect to the imaginary axis. The right half plane zeros indicate that the CPS-System would be very difficult to control if only  $z_p = [\mu \ x_2]^T$  were available for feedback. The fact that the right half plane zeros are close to right half plane poles exacerbates this matter [7].

### 2.3 Control Laws.

Several control laws are implemented within our environment. With our MATLAB integration capability, the user may seamlessly submit a model to MATLAB, generate a design, and then simulate, animate, etc. A user may also drive our animation module with a SIMULINK block diagram. This makes the environment highly extensible.

**Control System Structure.** In order to focus the exposition, this paper will restrict specific discussions to control laws having the structure indicated in Figure 3.

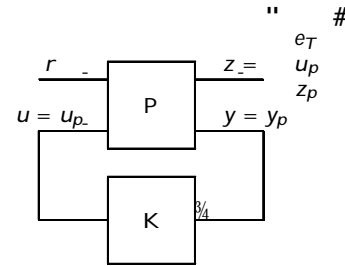


Figure 3: Control System Structure

In this Figure,  $P$  is the augmented plant model to be controlled,  $K$  is the MIMO compensator (to be designed) and  $e_T$  is the tracking error, defined as:

$$e_T \stackrel{\text{def}}{=} \begin{bmatrix} \mu_c - i & \mu_c \\ x_{2c} & i & x_2 \end{bmatrix} \quad (10)$$

**$H_1$  Design.** The controller  $K$  in Figure 3 was designed by solving the following weighted  $H_1$  optimization:

$$\begin{bmatrix} \circ \\ \circ \\ \circ \\ \circ \end{bmatrix} \circ \begin{bmatrix} W_{re_T} & T_{re_T} \\ W_{ru} & T_{ru} \\ W_{rzp} & T_{rzp} \end{bmatrix} \begin{bmatrix} \# \\ \circ \\ \circ \\ \circ \end{bmatrix} \cdot 1 \quad (11)$$

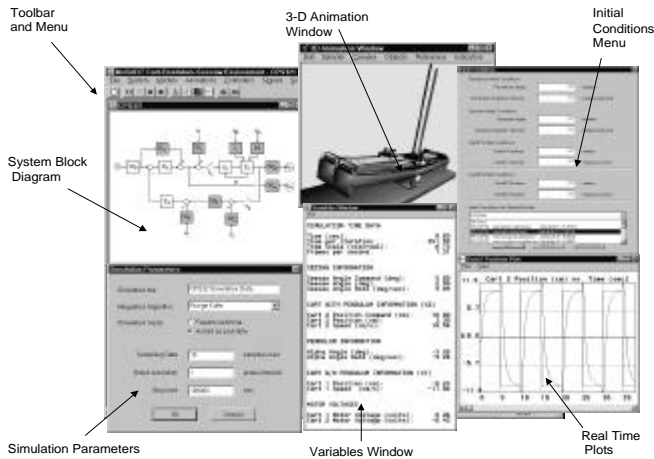


Figure 4: Program User Interface

with,

$$W_{reT} = \frac{h}{4E10^3} \frac{s+1}{s+10^4} i I_{2E2} \quad (12)$$

$$W_{ru} = 1.5E10^5 I_{2E2} \quad (13)$$

$$W_{rzp} = 1E10^5 s^2 I_{2E2} \quad (14)$$

$$\rho = 143 \quad (15)$$

#### 2.4 Advanced Models: MATLAB Integration

More advanced models may be defined in SIMULINK/MATLAB and integrated into the MoSART environment via the A-Lab feature. With A-Lab, the power and flexibility of MATLAB can be combined with the 3D animation capability of the Interactive Cart-Pendulum-Seesaw Environment.

### 3 Description of MoSART Cart-Pendulum-Seesaw Environment

Our Interactive MoSART Cart-Pendulum-Seesaw (CPS) Environment is an interactive application intended to serve as a virtual testbed to facilitate the analysis, design, and evaluation of high performance control laws for cart-pendulum-seesaw systems. The software runs on any PC-compatible computer running Microsoft Windows '95, '98, or NT. For optimum performance, a fast Pentium processor (300MHz+) and 3D-accelerated video card is recommended. We begin our discussion with some development background. The functionality and content of the individual modules that make up the environment are then described.

The Interactive MoSART Cart-Pendulum-Seesaw Environment is organized into four core modules: The Program User Interface Module (PIM), Simulation Module (SIM), Graphical Animation Module (GAM), and the Help/Instruct Module (HIM). Each of these modules is now discussed:

#### 3.1 Program User Interface Module (PIM)

The PIM provides the familiar Microsoft Windows interface to the user (Figure 4). Through this interface the

user can: select/edit a simulation model, select an animation model, select/edit the input signals of the simulation, view/change the simulation parameters, save/load the simulated data, or post-process the simulated data in MATLAB.

The active *child window* contains a block-diagram representation of the selected system. The user may edit parameters simply by clicking the mouse on the block diagram, or through the menus. Common functions are accessible through a floating/docking toolbar, which has a VCR-style control panel for controlling the simulation.

#### 3.2 Simulation Module (SIM)

The simulation engine numerically simulates a system which is based on a general block structure. The user can specify the use of different integration methods, control-laws, inputs, and other parameters of the system. User changes are accepted even as the simulation is progressing. More complex simulations may be developed to take advantage of direct access to MATLAB 5.0 scripts and toolboxes via the MATLAB engine. Faster than real-time simulations are possible on standard personal computers.

#### 3.3 Graphics/Animation Module (GAM)

The ability to visualize the simulation is a key feature of this environment. Several visual representations of the simulation are available to the user, including: real-time variable display windows, real-time graph plotting windows, and 3-dimensional animation windows (Figure 4). The 3D animations utilize Microsoft Direct-3D to provide a texture-mapped three-dimensional light-shaded polygon representation of the system. Direct-3D also allows the program to take advantage of any available 3D acceleration hardware on the computer platform.

#### 3.4 Help/Instruct Module (HIM)

This module allows for the inclusion of on-line tutorials and basic help information for using the environment. With direct links to Hypertext Markup Language (HTML) format documents, users can call up help and information directly from the environment. This allows the creation of detailed on-line model documentation, tutorials, model documentation, and project guidelines.

## 4 Educational Utility

The utility of the environment as a design, research, and education tool is now demonstrated.

#### 4.1 Modal Analysis

There are seven (7) natural modes associated with the Cart-Pendulum-Seesaw system being considered (about the equilibrium point  $x_p = 0$  and  $u = 0$ ). Two of these modes will be further described

Figure 5 shows how the CPS-System's *Pendulum Damping Mode* may be simulated from within the Interactive MoSART Cart-Pendulum-Seesaw Environment using the linear model for the plant.

The pendulum damping mode and the undamped seesaw oscillation mode are excited by setting the initial condition

of the system to the eigenvector (real part) associated with its respective mode (see 3.7 and 3.4 in Table 1 and turning  $\omega^0$  the inputs, allowing the systems's natural tendencies to evolve (notice that the actuator was turned  $\omega^0$  in the undamped seesaw oscillation mode). Figure 5 and Figure 6 indicated that the pendulum damping mode is a damped exponential mode and the seesaw oscillation mode is an undamped sinusoidal mode respectively. The instabilities shown in the figures are due to the (expected) numerical inaccuracies which occur within a finite precision simulation - inaccuracies which excite the system's natural toppling instabilities.

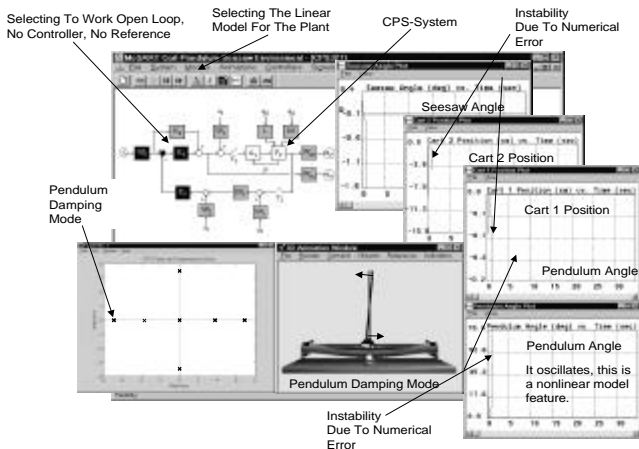


Figure 5: Visualization Of CPS-System's Pendulum Damping Mode On Linear Model Using Interactive MATLAB Environment

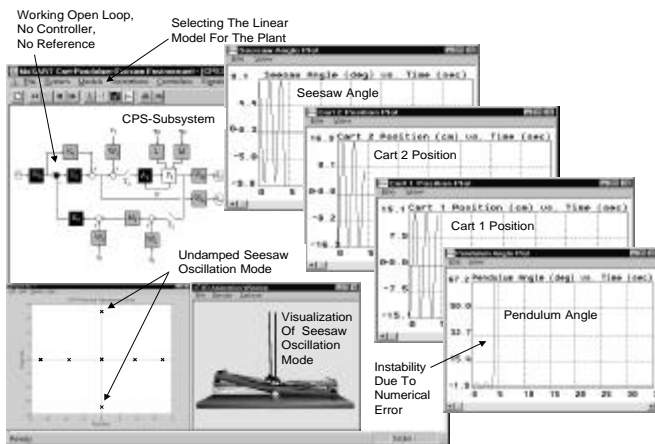


Figure 6: Visualization Of CPS-Subsystem's Undamped Seesaw Oscillation Mode On Linear Model Using Interactive MATLAB Environment

#### 4.2 Controller Design

Our Interactive MATLAB CPS Environment has been designed to accommodate control system design using weighted  $H^1$  optimization [16]. This is done using the environment's communication link with MATLAB. Users are permitted to alter

<sup>2</sup> three weighting functions  $W_{reT}$ ,  $W_{ru}$ ,  $W_{rzp}$ , and,

<sup>2</sup> a parameter  $\omega^0$  (the algorithm used searches for the optimal  $\omega^0$ )

These control system "design knobs" may be used to alter low frequency command following and disturbance rejection properties, control action, high frequency roll-off characteristics, and robustness properties. An example of such a procedure is shown in Figure 7. The weighting function of Section 2.3 were fed as shown. The resulting frequency responses for  $T_{reT}$  and  $T_{rzp}$  met the required design specifications.

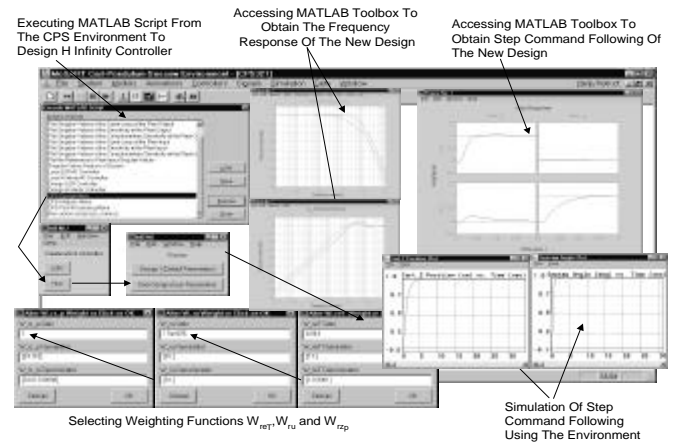


Figure 7:  $H^1$  Controller Design and Analysis Using CPS Environments' Link to MATLAB

#### 4.3 Linear Model Command Following

A square wave command is now used to evaluate the low frequency command properties of our previously designed  $H^1$  controller.

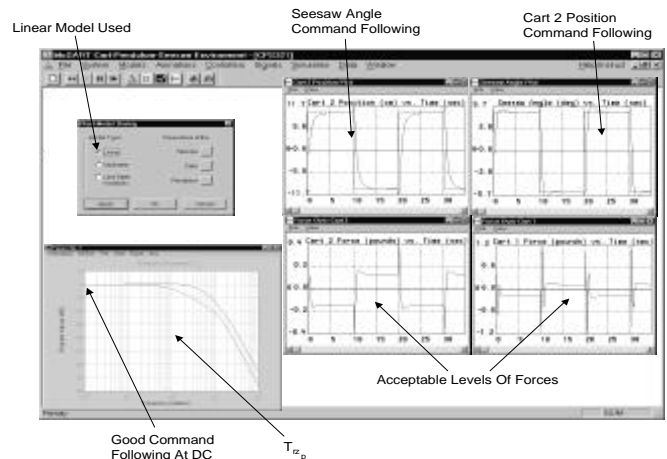
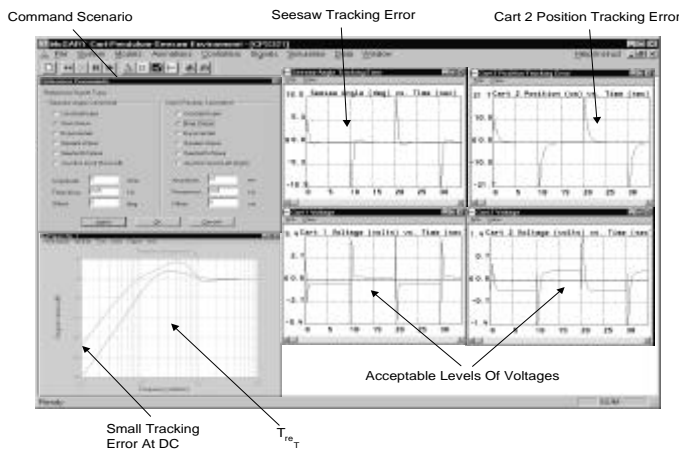


Figure 8: Low Frequency Command Following - Linear Plant,  $H^1$  Controller,  $r = [5 \text{ deg } 10 \text{ cm}]$  Square Wave,  $T = 20\text{sec}$ , Command Following,  $T_{rzp}$  Frequency Response, and Control Forces

Figure 8 and Figure 9 shows the response of the closed-loop system to a 10 cm square wave cart 2 position and 5

degrees seesaw angle command ( $T = 20$  seconds,  $f = 0.05$  Hz,  $\theta = 0.31$  rad/sec). The frequency response for  $T_{reT}$  and  $T_{rzp}$  are included to support our discussion.



**Figure 9:** Low Frequency Command Following - Linear Plant,  $H^1$  Controller,  $r = [5 \text{ deg } 10 \text{ cm}]$  Square Wave,  $T = 20$  sec, Tracking Error,  $T_{reT}$  Frequency Response and Control Voltages

The resulting cart 2 position and seesaw angle response is seen to follow the reference command well with

- little overshoot (less than 10%) and a
- settling time of about  $t_s \approx \frac{5}{BW} \approx \frac{5}{0.8} \approx 6.25$  seconds.

The good steady state command following is expected from the frequency responses for  $T_{reT}$  and  $T_{rzp}$ . The observed 5% transient undershoot is a consequence of the plant's right half plane (non-minimum phase) zero in the plant transfer function associated with the cart 2 position and seesaw angle. Physically, cart 2 and the seesaw initially moves opposite to the command (also see initial applied voltage or force) in order to get the pendulum and the seesaw "falling" in the right direction - the direction of the command. The corresponding control voltages (actuator input) and control forces (actuator output) responses are seen to be acceptable.

## 5 Summary and Future Directions

This paper has described a software environment for simulating and visualizing cart-pendulum-seesaw related systems. Advanced visualization aids are used to abstract out critical behavior. A user friendly interface permits a user to alter the seesaw model, cart model, pendulum model, actuator model, other model parameters, control law, control law parameters, control law design parameters for control law redesign via MATLAB engine and toolboxes, reference commands, disturbances, shaping filters, integration routines, and other signal and system parameters. A modal analysis of the cart-pendulum-seesaw (CPS) system, an interactive controller design and an  $H_1$  controller design evaluation were done to illustrate the utility of the environment as a research and education tool. Future work will

include the incorporation of additional MIMO control laws (e.g.  $H_1$ -synthesis, LQG/LTR, LQR, gain scheduling, etc). The models will be further developed to include actuator saturation and deadzones. Some of this work has already been initiated.

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