Communications and Signal Processing
Fall 2008 MSE Exam

Name:____

Please put your name on each page of this test.

This test is composed of **two parts**. You must work **four** questions from Part 1, and **three** questions from Part 2. If you answer more than seven questions, we will choose randomly which seven to correct. So, please mark below those problems that you wish to have graded. Also, please write your name on each page as different professors grade different questions of the examination.

Part 1 (work 4 problems)

______ 1
______ 2
______ 3
______ 4
______ 5
______ 6

Part 2 (work 3 problems)

______ 7
______ 8
______ 9
______ 10
______ 11
______ 12
______ 13
______ 14

You may use a calculator, a book of math tables (e.g. CRC tables) and tables of Fourier transform pairs, but no other reference material. The test is three hours long.

1. **EEE 203–Signals and Systems**

Suppose we have a linear, time-invariant continuous-time system with a real-valued impulse response. Let the Fourier Transform of the impulse response be given by $H(j\omega)$. Suppose $x(t) = \cos(\omega_0 t)$ is the input to this system.

(a) Express the output $y(t)$ in terms of $H(j\omega)$ and ω_0.

(b) Is the output periodic? If so, with what period?

(c) What are the Fourier Series coefficients of the output $y(t)$?

2. **EEE 350–Random Signal Analysis**

The duration of a cellular phone call is an exponential random variable with expectation 3 minutes. A subscriber has a calling plan that includes 300 minutes per month at a cost of \$30, plus \$0.2 for each minute that the total calling time exceeds 300 minutes. In a certain month the subscriber has made 100 independent cellular calls.

(a) Assume that the telephone company measures the call duration exactly without rounding up fractional minutes, and charge accordingly. Use the central limit theorem to estimate the probability that the subscriber's bill is greater than \$36.

(b) Suppose the company does charge a full minute for each fractional minute used. Re-calculate your estimate of the probability that the bill is greater than \$36.

3. **EEE 407–Digital Signal Processing**

For the filter below:

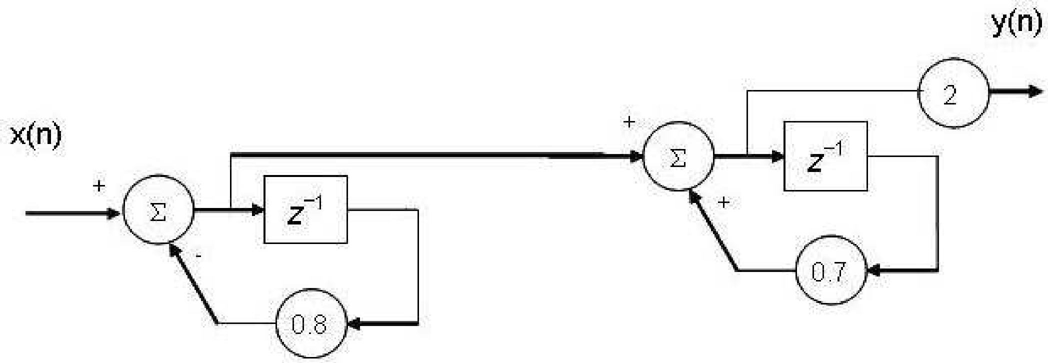
(a) Write a single difference equation for the entire system with $x(n)$ as the input and $y(n)$ as the output.

(b) Determine the impulse response of the entire system in closed form

(c) Find the steady state response of the filter to $x(n) = (-1)^n + 4\sin(\pi n/3)$

(d) Determine the poles and zeros of this entire filter and place them on the z plane.

(e) Determine and sketch a parallel implementation of this filter with all parameters stated.



4. **EEE 407–Digital Signal Processing**

Let us represent the Discrete Fourier Transform (DFT) by a matrix $\mathbf{F}$

(a) Define precisely the entries of $\mathbf{F}$ and $\mathbf{F}^{-1}$ for $N = 4$.

(b) Lets us consider the 4×1 vector $\mathbf{x} = [8\ 0\ 2\ 0]^T$. Compute $\mathbf{X} = \mathbf{F}\mathbf{x}$

(c) Show how the same computation can be done with two 2-point *DFTs*. Can we generalize this idea for N point *DFTs* and under what conditions?

(d) Form a diagonal 4×4 matrix $\mathbf{X}_d$ whose diagonal entries are identical with those in the vector $\mathbf{X}$. Determine the matrix $\mathbf{x}_c$ where $\mathbf{x}_c = \mathbf{F}^{-1}\mathbf{X}_d\mathbf{F}$ without performing explicitily this calculation and state clearly why the calculation is not necessary to define $\mathbf{x}_c$.

(e) What is the relationship of the entries of the vector $\mathbf{X}$ and the matrix $\mathbf{x}_c$.

(f) Explain how the relation $\mathbf{x}_c = \mathbf{F}^{-1}\mathbf{X}_d\mathbf{F}$ can be exploited for fast convolution for an $N \times 1$ signal vector $\mathbf{x}$ and another $N \times 1$ vector $\mathbf{h}$.

5. **EEE 455–Communication Systems**

We have an alphabet of three letters $X = \{a, b, c\}$ with probabilities $p_a = 0.2$, $p_b = 0.3$, and $p_c = 0.5$.

(a) Find the binary entropy of X

(b) Find the average codeword length of the Huffman code for X

(c) Find the Huffman codes for the 9 combinations of two letters with independent probabilities for the letters, i.e. $p_{aa} = 0.04$, $p_{ab} = 0.06$, $p_{ac} = 0.1$, etc.

6. **EEE 455–Communication Systems**

   What is a superheterodyne receiver? Why is it used? Make sure to give a block diagram with your explanation. Also explain the concept of “image frequency”, and give as much detail as possible for the RF filter (amplifier) and IF filter (amplifier) that will be used.

7. **EEE 552–Digital Communications**

Consider a channel with the input output relationship $y = x + zn$ where x is the channel input and y is the channel output, z is a binary random variable independent of the input taking on the values "1" or "2" with probability 1/2 each, and n is zero mean white Gaussian noise with variance σ^2. The effect of the random variable z is clearly to make the noise variances for different uses of the channel different in a random manner.

Assume that BPSK modulation is used with equally likely bits, i.e., $x = 1$ or $x = -1$ with probability 1/2 each. The receiver observes y and tries to make a decision on x only.

(a) Derive the optimal decision rule.

(b) What is the resulting probability of error?

8. **EEE 554–Random Signal Theory**

Consider a Poisson process in which N_τ is the number of occurrences of an activity (e.g. the arrivals of messages at a digital communication multiplex) over an time interval $(t, t+\tau]$ and N_τ follows a Poisson distribution, that is

$$P(N_\tau = k) = \begin{cases} e^{-\lambda\tau}\frac{(\lambda\tau)^k}{k!}, & k \in \mathcal{Z}, k \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda > 0$. Assume that at one time instant, there can be at most one occurrence of the activity.

Let T be the random variable representing the time interval between two successive occurrences of the activity in this Poisson process.

Show that T follows an exponential distribution. Find the parameter of this exponential distribution.

9. **EEE 459/591–Communication Networks**

Suppose two hosts, A and B, are attached to opposite ends of a 1.5 Km cable, and that they each have one packet of 2000 bits (including all headers and preambles) to send to each other. Both hosts attempt to transmit at time $t = 0$. Suppose there are three hubs (repeaters) between A and B, each inserting a 30 bit delay. Assume the transmission rate is 10 Mbps, and CSMA/CD with backoff intervals of multiples of 512 bits is used. After the first collision, A draws $K = 0$ and B draws $K = 1$ in the exponential backoff protocol. Ignore the jam signal.

(a) What is the one–way propagation delay (including repeater delays) between B and A in seconds. Assume that the signal propagation speed is 2×10^8 m/sec.

(b) At what time in seconds is B's packet completely delivered at A.

(c) Now suppose that only B has a packet to send and that the repeaters are replaced with switches. Suppose that each switch has a 30 bit processing delay in addition to a store-and-forward delay. At what time in seconds is B's packet delivered at A?

10. **EEE 507–Multidimensional Signal Processing**

Consider a bandlimited continuous-domain signal $x_c(t_1, t_2)$. The signal $x_c(t_1, t_2)$ is sampled, and the resulting samples are located at the points $(n_1 + 2n_2, n_1)$ in the (t_1, t_2) domain, where $-\infty \leq n_1, n_2 \leq \infty$ and n_1, n_2 integers.

(a) Sketch the resulting sampling lattice (show at least 6 sample locations).

(b) Determine a sampling matrix V that generates the resulting sampling lattice, and determine the resulting sampling density.

(c) Assume that the spectrum $X_c(\Omega_1, \Omega_2)$ of $x_c(t_1, t_2)$ is non-zero over the rectangular-shaped region that includes the frequency point (0,0) and whose boundaries are delimited by the following 4 line segments:

$$\Omega_1 = \frac{\pi}{4} \quad (1)$$

$$\Omega_2 = \frac{\pi}{4} \quad (2)$$

$$\Omega_1 = -\frac{\pi}{4} \quad (3)$$

$$\Omega_2 = -\frac{\pi}{4} \quad (4)$$

where Ω_1 and Ω_2 are in radians/m. Sketch the spectrum of the *sampled* signal $x(n_1, n_2)$ and determine whether the signal $x_c(t_1, t_2)$ can be reconstructed from the sampled signal $x(n_1, n_2)$ (Justify your answer).

11. **EEE 558–Wireless Communications**

Recall that the probability of error for binary orthogonal signaling with noncoherent detection is given by

$$P_e(\gamma_b) = \frac{1}{2} e^{-\gamma_b/2} \ ,$$

where γ_b is the SNR per bit (i.e., $\gamma_b = E_b/N_0$). In wireless communications, due to the random fluctuations of the channel, the SNR per bit is attenuated by a random variable r, which is often assumed to have an exponential distribution so that the pdf of r is given by $p(r) = e^{-r}, r > 0$. Express the *average* probability of error given by

$$\overline{P}_e(\gamma_b) = \int_0^\infty P_e(r\gamma_b)p(r)dr \ .$$

We may interpret $P_e(\gamma_b)$ as the performance over the AWGN channel with no fading, and $\overline{P}_e(\gamma_b)$ as the performance over the wireless fading channel.

a) Based on the expression for $\overline{P}_e(\gamma_b)$, is the performance over the AWGN better, or the performance in the presence of fading, at high SNR? Why?

b) Explain the notion of "diversity", and comment on how employing a form of diversity would effect the performance over the AWGN channel, and performance over the fading channel, separately.

12. **EEE 508–Digital Image Processing and Compression**

Consider a vector quantizer (VQ) with a codebook consisting of 256 codevectors, and consider fixed-length coding of the quantization indices.

(a) Determine the codevector length that is needed to achieve a bit-rate of 3 bits per pixel.

(b) If the 128-entry VQ codebook was designed using a binary splitting algorithm and is the last stage of a tree-structured VQ (TSVQ), determine the number of stages of the corresponding TSVQ.

(c) Let the 128-entry VQ be implemented as an M-stage residual vector quantizer (RVQ) with a 2-entry codebook at each stage. Determine M. Justify your answer.

(d) If each scalar value in the codebooks is representing as a byte (8 bits), determine the storage requirement in bytes for the TSVQ of (b) and RVQ of (c).

13. **EEE 556–Detection and Estimation**

Consider the problem of detecting a signal $s[n]$ under two hypothesis, H_0 (signal absent) and H_1 (signal present). The observations $x[n]$ are given as

$$\begin{aligned} H_1 : \quad x[n] &= s[n] + w[n], \quad n = 0, \ldots, N-1 \\ H_0 : \quad x[n] &= w[n], \quad n = 0, \ldots, N-1 \end{aligned}$$

where $s[n]$ is a zero-mean, white Gaussian sequence with variance σ_s^2 and $w[n]$ is a zero-mean white Gaussian sequence with variance σ_w^2.

Derive the optimal Neyman-Pearson detector to decide between H_0 and H_1 based on the N observation samples.

14. **EEE 505–Time-Frequency Signal Processing**

The Wigner distribution (WD) of a signal $x(t)$ is defined as

$$\mathrm{WD}_x(t,f) = \int_\tau x\left(t+\frac{\tau}{2}\right) x^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi\tau f}\, d\tau$$

(a) Show whether the WD preserves scale changes on the analysis signal.

(b) Compute the WD of the sum of two sinusoids, $x(t) = e^{j2\pi f_1 t} + e^{j2\pi f_2 t}$ where $f_1 < f_2$. Discuss the location of the cross terms in the time-frequency plane.

(c) What will happen to the cross terms if we now use the WD to analyze the compresed signal $y(t) = x(2t)$? Explain your answer.