

Communications and Signal Processing
Spring 2008 MSE Exam

Name: _____

Please put your name on each page of this test.

This test is composed of **two parts**. You must work **four** questions from Part 1, and **three** questions from Part 2. If you answer more than seven questions, we will choose randomly which seven to correct. So, please mark below those problems that you wish to have graded. Also, please write your name on each page as different professors grade different questions of the examination.

Part 1 (work 4 problems)

_____ 1
_____ 2
_____ 3
_____ 4
_____ 5
_____ 6

Part 2 (work 3 problems)

_____ 7
_____ 8
_____ 9
_____ 10
_____ 11
_____ 12
_____ 13
_____ 14

You may use a calculator, a book of math tables (e.g. CRC tables) and tables of Fourier transform pairs, but no other reference material. The test is three hours long.

1. **EEE 203–Signals and Systems**

Consider the following relationship between the input $x[\cdot]$ and the output $y[\cdot]$ of a system:

$$y[n] = \sum_{k=n-1}^{n+1} x[k]$$

Is this system (i) linear, (ii) time-invariant, (iii) causal, (iv) stable? Please justify your answer in each case.

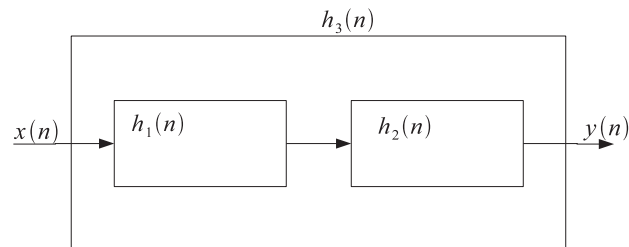
2. EEE 350–Random Signal Analysis

In a communication system, information bits are transmitted from source to destination. However, due to the ambient white Gaussian noise in the communication channel, an information bit may be received erroneously by the time it arrives at the destination. Assume that the information bits are transmitted independently, and let p denote the bit error probability with $p \in (0, 1)$. Suppose a total of N bits are transmitted from the source to the destination, and let W_N denote the total number of bits received erroneously.

- a) What is the exact probability mass function (PMF) of W_N ? Find the expectation and variance of W_N .
- b) Suppose $p = 10^{-3}$ and $N = 10^7$. Use the Central Limit Theorem to estimate the probability $P(W_N \geq 10, 150)$.

3. EEE 407–Digital Signal Processing

Two digital filters with impulse responses $h_1(n) = (0.9)^n u[n]$ and $h_2(n) = (-0.9)^n u(n)$ are connected as shown below. The overall filter has impulse response $h_3(n)$ ($h_3(n)$ is the overall impulse response corresponding to the system below).



- Determine the overall impulse response of the entire filter in closed form
- Give one difference equation that describes the overall system ($h_3(n)$) and specify the coefficients of the filter.
- Determine and sketch carefully the poles and zeros.
- Sketch carefully its frequency response
- Determine the steady state response of the overall filter to the input $x(n) = u(n) + 2 \sin(\pi n/2)$

4. **EEE 407–Digital Signal Processing**

Consider a discrete-time, linear time-invariant filter $H(z)$, $|z| > 0.8$, whose zero locations are given at $z = e^{j3\pi/4}$, $z = e^{j7\pi/8}$, and pole locations at $z = 0.8$, $z = 0.8e^{j\pi/8}$ in the z-plane.

- (a) Sketch the pole-zero diagram of $H(z)$ in the z-plane.
- (b) Write down an expression for $H(z)$ (upto a gain constant) and sketch the corresponding magnitude response $|H(e^{j\omega})|$ directly from the pole-zero plot.
- (c) What type of filter is $H(z)$? Explain your answer.
- (d) Is this a stable filter? Explain your answer.

5. **EEE 455–Communication Systems**

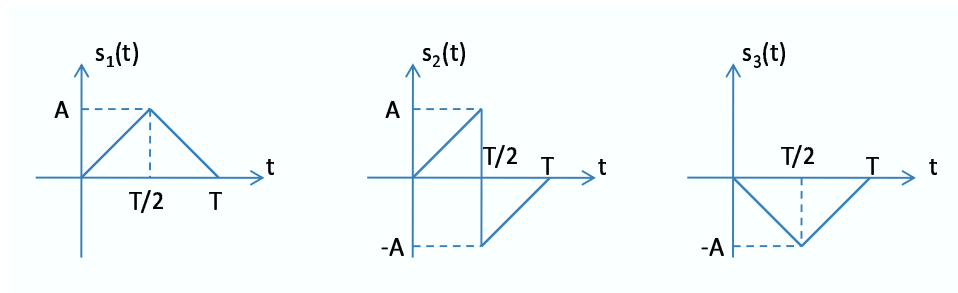
Consider MFSK with signals

$$s_m(t) = \sqrt{\frac{2E}{T}} \cos(2\pi(f_c + \frac{m}{2T})t) \quad m = 0, 1, 2, \dots, M - 1$$

- a. Verify in fact that the signals are orthogonal to each other and therefore form a basis. (For example, integrate $s_0(t)s_1(t)$ over $[0, T]$).
- b. Draw the signal points for 2FSK and the associated decision region for each point.
- c. Suppose we have a noise energy per bit that is 10 dB less than the signal power. Find the bit error rate.

6. **EEE 455–Communication Systems**

A ternary communication system is using the three pulses shown below for communication. That is, to transmit the first message $s_1(t)$ is used, and so on. All the three possible symbols are independent and equally likely, and the channel is additive white Gaussian noise (AWGN) channel.



- a) Find a set of orthonormal basis functions for this signal set, and show the signal constellation.
- b) Describe the matched filter type optimal receiver structure. Give a block diagram and be specific.

7. EEE 552–Digital Communications

A communication system employs M signals $\{s_m(t)\}_{m=1}^M$ for transmission of M equiprobable messages. The receiver has two antennas, and it receives two copies of the signals: $r_1(t) = s_m(t) + n_1(t)$ and $r_2(t) = s_m(t) + n_2(t)$ by these antennas. Both $n_1(t)$ and $n_2(t)$ are white Gaussian noises with power spectral densities $N_{01}/2$ and $N_{02}/2$, respectively. The receiver makes its optimal detection based on the observations of both $r_1(t)$ and $r_2(t)$. It is further assumed that the two noise processes are independent.

What is the optimal decision rule at the receiver. Show the details of your work to receive credit.

8. EEE 554–Random Signal Theory

A discrete-time WSS stochastic process $\mathbf{x}(\cdot)$ with mean $\mu_{\mathbf{x}} = -1$ and autocovariance function $C_{\mathbf{x}}(\kappa) = 2^{-|\kappa|}$ is passed through a LTI filter with impulse response $h(k) = \delta(k) - \delta(k - 1)$. Calculate the mean $\mu_{\mathbf{y}}$ and (power) spectral density $S_{\mathbf{y}}(\omega)$ of the output process $\mathbf{y}(\cdot)$.

9. **EEE 459/591–Communication Networks**

Suppose you are downloading a web page consisting of a base HTML page and n embedded objects using different forms of HTTP. Let RTT (in seconds) denote the round trip time between your host and the web server. Suppose that the base page and each of the embedded objects have size F (in bit) and the transmission rate over a TCP connection is R (in bit/sec).

a) Suppose that persistent HTTP without pipelining is used. How long does it take to download the web page?

b) Suppose that persistent HTTP with pipelining is used. How long does it take to download the web page?

c) Suppose that non-persistent HTTP is used. How long does it take to download the web page?

d) Suppose that non-persistent HTTP with k parallel connections is used. How long does it take to download the web page? You may assume that n/k is an integer.

10. **EEE 507–Multidimensional Signal Processing**

Consider the following 2-D discrete-domain signals:

$$x(n_1, n_2) = \begin{cases} 1, & 0 \leq n_1 \leq 2 \text{ and } 0 \leq n_2 \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

and

$$y(n_1, n_2) = \begin{cases} \sum_{r_1=-\infty}^{\infty} \sum_{r_2=-\infty}^{\infty} x(n_1 - 2r_1, n_2 - 2r_2), & 0 \leq n_1 \leq 1 \text{ and } 0 \leq n_2 \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- a. Determine the DTFT of $x(n_1, n_2)$.
- b. Plot $y(n_1, n_2)$.
- c. Let $Y(K_1, K_2)$ be the 2×2 -point DFT of $y(n_1, n_2)$. Express $Y(K_1, K_2)$ in terms of the DTFT of $x(n_1, n_2)$.
- d. Determine $Y(K_1, K_2)$ the 2×2 -point DFT of $y(n_1, n_2)$.

11. **EEE 558–Wireless Communications**

Consider the following single-input, multiple-output system with the following input/output relationship:

$$x_m = \sqrt{\rho} h_m s + v_m \quad m = 1, \dots, M$$

where x_m is the sampled output, $s \in \{-1, +1\}$ is the transmitted BPSK symbol; h_m is the complex Gaussian channel gain with zero mean and unit variance, and v_m is the complex-valued noise sample with zero-mean and unit one at the m^{th} diversity branch; ρ is the average SNR.

- a. Given $x_m, m = 1, \dots, M$, what is the optimum decision rule?
- b. Derive the average probability of error as a function of ρ .
- c. What is the diversity order? Why?

Hint: To simplify your analysis, you can assume that $Q(x) \approx \frac{1}{2} \exp(-x^2/2)$.

12. **EEE 508–Digital Image Processing**

Consider an image intensity x which can be modeled as a sample whose probability density function is

$$f_X(x) = \frac{3}{16} \delta(x - 1) + \frac{5}{16} \delta(x - 2) + \frac{1}{4} \delta(x - 3) + \frac{1}{4} \delta(x - 4)$$

- a. If x is quantized using a scalar quantizer with decision levels $\{t_1 = 0; t_2 = 2.5; t_3 = 5\}$ and reconstruction levels $\{r_1 = 1; r_2 = 4\}$. Compute the resulting MSE and the entropy of the quantized output.
- b. Consider the following 4×4 image block B

1	1	2	3
1	2	3	4
2	2	3	4
2	3	4	4

Using the scalar quantizer described in part a, compute the quantized output block $\hat{B} = Q(B)$.

- c. If the 4×4 block B of (b) is quantized using a vector quantizer, what are the possible codevector sizes N that can be used to quantize B ? For each possible codevector size N , determine the maximum number of codebook entries so that the achieved bit-rate ≤ 0.5 bit per sample.

13. **EEE 557–Broadband Networks**

Consider a packet queue for which packets arrives as a Poisson process at a rate of λ packets per second, while packets are transmitted at a rate of μ packets per second, with packet transmission time exponentially distributed with mean $1/\mu$. There is a waiting room for B packets in the queue, including the one being transmitted. In essence, the queue is an M/M/1/B queue.

- a. Draw the Markov chain for the states of the queue
- b. If $\lambda = 5$, $\mu = 10$, and $B = 4$, what is the probability that a packet arriving would be lost when the waiting room is full?
- c. For the parameters in part b, what is the average time in the system for a packet that is not lost?

14. **EEE 505–Time-Frequency Signal Processing**

The spectrogram (SPEC) is defined as the squared-magnitude of the short-time Fourier transform (STFT). Using an analysis window $h(t)$, the STFT is given by:

$$\text{STFT}_x(t, f) = \int_{\tau} x(\tau) h^*(\tau - t) e^{-j2\pi f\tau} d\tau$$

- (a) Discuss a possible filter implementation of the STFT.
- (b) Assume that you want to analyze a signal consisting of two sinusoids of different frequencies. Discuss what type of window you would use in the spectrogram in order to best resolve the sinusoids.
- (c) Demonstrate whether or not the spectrogram preserves time shifts on the analysis signal. That is, show whether the following relationship holds:

$$y(t) = x(t - t_0) \quad \Rightarrow \quad \text{SPEC}_y(t, f) = \text{SPEC}_x(t - t_0, f)$$

