Communications and Signal Processing
Spring 2009 MSE Exam

Name:____

Please put your name on each page of this test.

This test is composed of **two parts**. You must work **four** questions from Part 1, and **three** questions from Part 2. If you answer more than seven questions, we will choose randomly which seven to correct. So, please mark below those problems that you wish to have graded. Also, please write your name on each page as different professors grade different questions of the examination.

Part 1 (work 4 problems)

______ 1
______ 2
______ 3
______ 4
______ 5
______ 6

Part 2 (work 3 problems)

______ 7
______ 8
______ 9
______ 10
______ 11
______ 12
______ 13
______ 14
______ 15
______ 16

You may use a calculator, a book of math tables (e.g. CRC tables) and tables of Fourier transform pairs, but no other reference material. The test is three hours long.

1. **EEE 203–Signals and Systems**
   Consider a continuous-time signal: $x(t) = 1$, if $0 \leq t \leq T$, and $x(t) = 0$ if t is not in the above interval. In other words, $x(t)$ is a pulse of width T and height 1. Suppose also that $h(t) = x(t)$.

   (a) Calculate the convolution $y(t) = x(t) * h(t)$. *Show your work.*

   (b) Plot $y(t)$ versus t, *carefully labeling your plot.*

2. **EEE 350–Random Signal Analysis**
   In a digital communication system, for each transmission, a transmitter sends one of the two possible symbols (1 or 0) to a receiver. Let B_1 denote the event "the transmitted symbol is 1" and B_2 denote the event "the transmitted symbol is 0". Furthermore, let A_1 denote the event "the received symbol is determined to be 1" and A_2 denote the event "the received symbol is determined to be 0". Then, $P(A_1|B_1) = P(A_2|B_2) = 0.8$, and $P(A_2|B_1) = P(A_1|B_2) = 0.2$. Suppose that $P(B_1) = 0.6$ and $P(B_2) = 0.4$. Find the probabilities $P(B_1|A_1)$ and $P(B_2|A_1)$.

3. **EEE 407–Digital Signal Processing**
Two causal filters with impulse responses $h_1(n) = 0.9^n u(n) + 0.9^{n-1}u(n-1)$ and $h_2(n) = (-0.7)^n u(n)$ are connected as shown below. The overall filter has impulse response $h_3(n)$ (i.e., $h_3(n)$ is the overall impulse response corresponding to the system boxed with the dotted line).

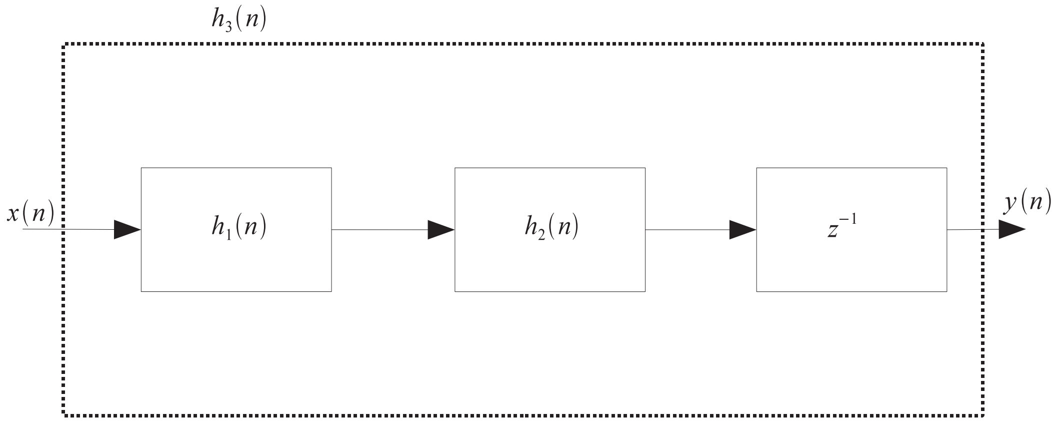


Figure 1: System Model.

(a) Find the poles and zeros of this system and place them on the z-plane.

(b) Sketch the magnitude frequency response function of this system.

(c) Find the overall impulse response of the system.

(d) Give a memory-efficient realization of the system.

4. **EEE 407–Digital Signal Processing**
Consider the following discrete-time Fourier transform (DTFT) pair:

$$a^{|n|} \quad \longleftrightarrow \quad \frac{1-a^2}{1-2\,a\,\cos\omega + a^2}\,, \quad |a|<1\,, \quad |\omega|<\pi$$

(a) Find the DTFT of $x[n] = (\frac{1}{3})^{|n|}$.

(b) Find the DTFT of $r[n] = 6(\frac{1}{3})^{|n-2|}$.

(c) Find the signal $g[n]$ whose DTFT is given by $G(e^{j\omega}) = (\frac{1}{3})^{|\omega|}$

5. **EEE 455–Communication Systems**

   (a) Plot the Fourier Transforms for $c(t) = A\cos 2\pi f_c t$ and for $m(t) = \frac{\sin \pi f_m t}{\pi f_m t}$

   (b) Also plot the spectrum of the AM modulated signal $u(t) = Am(t)\cos 2\pi f_c t$. What is the energy of the AM modulated signal?

   (c) Now suppose we use Phase Modulation with $k_p = 10$, what is the bandwidth of the PM signal?

   (d) Now suppose we use Frequency Modulation with $k_f = 20$, what is the bandwidth of the FM signal?

   (e) Describe how you would obtain SSB-SC from $u(t)$, and plot the corresponding spectrum.

6. **EEE 455–Communication Systems**
   Let the message signal be $m(t) = \frac{1}{2}[\text{sinc}(1000t) + \text{sinc}^2(1000t)]$.

   (a) What is $M(f)$? What is the bandwidth of the message signal?

   (b) Assume that the carrier is $c(t) = \sin(200000\pi t)$ and conventional AM is used, i.e., $u(t) = (1 + m(t))\sin(2\pi f_c t)$. What is $U(f)$? Plot the magnitude and the phase of $U(f)$ separately. What is the modulation index?

   (c) If an envelope detector is used to demodulate the signal in part b, what should the time constant of the RC circuit should be?

7. **EEE 509–DSP Algorithms and Software**

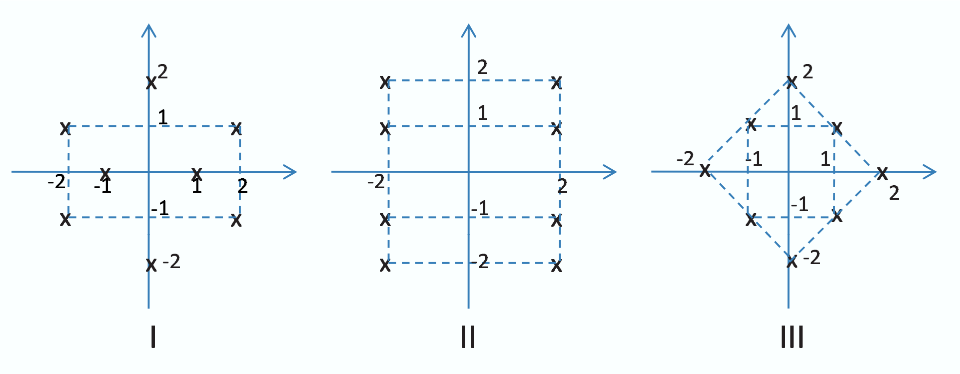
For the following impulse response sequence

$$h(n) = u(n) - u(n-4) \tag{1}$$

(a) Give the z transform as a rational function and state and sketch clearly the region of convergence with poles/zeros as needed.

(b) Give the steady state response due to $\cos(\pi n)$.

8. **EEE 552–Digital Communications**
Consider the 8-ary signal constellations below. Assume that they are used over an AWGN channel.

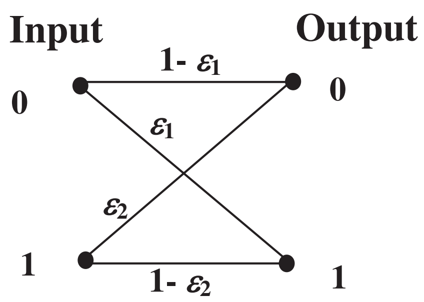


How would you compare the performance of these constellations in terms of their power efficiencies at high SNRs? Be quantitative, that is, rank order them, and specify the difference between their performance (in terms of the required SNRs for a given error probability in dBs).

9. **EEE 554–Random Signal Theory**
   A nonsymmetric binary communication channel is shown below. Assume that the inputs are equiprobable.

   (a) Find the probability that the output is 0.

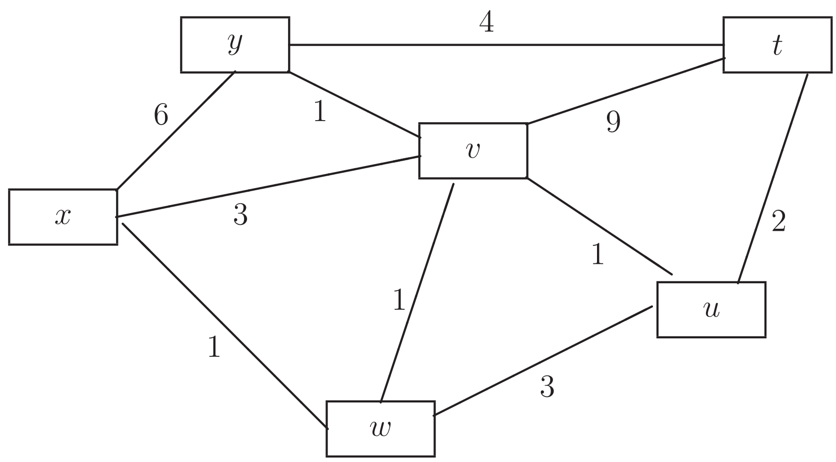
   (b) Find the probability that the input was 0 given the output is 1. Find the probability that input was 1 given that the output is 1. Which input is more probable given the output is 1?



10. **EEE 459/591–Communication Networks**
Consider the network shown in the figure below. Use Dijkstra's shortest-path algorithm to compute the shortest paths from node t to all other network nodes.

(a) Show how the algorithm works by tabulating known shortest distances and predecessor nodes for each network node for each step of the algorithm. In the table please assign the nodes to the columns in alphabetical order, i.e., arrange the columns in the order u, v, w, x, and y from left to right. When you encounter ties in executing the algorithm, please break the ties in favor of the leftmost column.

(b) Based on the results from Dijkstra's algorithm give the forwarding table at node t, i.e., for each destination node give the next node along the shortest path toward the destination.



11. **EEE 507–Multidimensional Signal Processing**
Consider a bandlimited continuous-domain signal $x_c(t_1, t_2)$. The signal $x_c(t_1, t_2)$ is sampled, and the resulting samples are located at the points $(n_1 + 2n_2, n_1 - n_2)$ in the (t_1, t_2) domain, where $-\infty \leq n_1, n_2 \leq \infty$ and n_1, n_2 integers.

   (a) Sketch the resulting sampling lattice (show at least 6 sample locations).

   (b) Determine a sampling matrix V that generates the resulting sampling lattice, and determine the resulting sampling density.

   (c) Assume that the spectrum $X_c(\Omega_1, \Omega_2)$ of $x_c(t_1, t_2)$ is non-zero over the diamond-shaped region that includes the frequency point (0,0) and whose boundaries are delimited by the following 4 line segments:

$$\Omega_2 = \Omega_1 + \frac{\pi}{3} \quad (2)$$

$$\Omega_2 = \Omega_1 - \frac{\pi}{3} \quad (3)$$

$$\Omega_2 = -\Omega_1 + \frac{\pi}{3} \quad (4)$$

$$\Omega_2 = -\Omega_1 - \frac{\pi}{3} \quad (5)$$

   where Ω_1 and Ω_2 are in radians/m. Sketch the spectrum of the *sampled* signal $x(n_1, n_2)$ and determine whether the signal $x_c(t_1, t_2)$ can be reconstructed from the sampled signal $x(n_1, n_2)$ (Justify your answer).

12. **EEE 556–Detection and Estimation**
Consider the problem of detecting a signal $s[n]$ under two hypothesis, H_0 (noise-only) and H_1 (signal plus noise). The observation $x[n]$ under each hypothesis is given by:

$$\begin{aligned} H_0 : \ x[n] &= w[n], \ n = 0, \ldots, N-1 \\ H_1 : \ x[n] &= s[n] + w[n], \ n = 0, \ldots, N-1 \end{aligned}$$

where $s[n]$ is a deterministic known signal and $w[n]$ is a zero-mean, white Gaussian noise sequence with **unknown** variance σ^2.

(a) Find the maximum likelihood estimate of the noise variance under each hypothesis.

(b) Obtain the Neyman-Pearson detector, given a probability of false alarm P_{FA}.

13. **EEE 558–Wireless Communications**
Assume that BPSK signaling is used to transmit digital data over the channel described by

$$r_l(t) = \alpha b_1 s_l(t) + z(t)$$

where b_1 is the binary information bit, $r_l(t)$ is the received low-pass equivalent signal, $s_l(t)$ is the transmitted low-pass equivalent signal with energy E_b, and $z(t)$ is low-pass white Gaussian noise with power spectral density N_0 (i.e., the noise variance is $N_0/2$ and the transmitter SNR is $2E_b/N_0$). Suppose that the channel gain α has the following probability mass function (PMF)

$$P(\alpha) = \begin{cases} 0.8, & \text{if } \alpha = \sqrt{0.6}, \\ 0.2 & \text{if } \alpha = \sqrt{0.3} \end{cases}$$

In this problem, we are interested in the outage probability, defined as follows:

$$P\{\text{SNR } \leq E_b/N_0\}$$

Suppose that the two-branch antenna diversity is used with selection combining. Suppose that the two branches are independent, i.e., the fading coefficients α_1 and α_2 are independent. Determine the corresponding outage probability. Now suppose that the two-branch antenna diversity is used with maximum ratio combining; determine the corresponding outage probability.

14. **EEE 508–Digital Image Processing**
Consider a two-dimensional, linear and shift-invariant, system with the following impulse response:

$$h(n_1,n_2) = \begin{cases} -2, & (n_1,n_2) = (0,0) \\ -4, & (n_1,n_2) = (1,0) \\ -1, & (n_1,n_2) = (2,0) \\ 1, & (n_1,n_2) = (0,2)\&(1,1) \\ 0.5 & (n_1,n_2) = (0,1)\&(2,2) \\ 0.25, & (n_1,n_2) = (2,1) \\ 2, & (n_1,n_2) = (1,2) \\ 0, & \text{otherwise} \end{cases}$$

Consider a 3×3 subimage $x(n_1,n_2)$ given by:

$$x(n_1,n_2) = \begin{cases} 20, & n_1 = 0 \text{ and } 0 \le n_2 \le 2 \\ 40, & n_1 = 1 \text{ and } 0 \le n_2 \le 2 \\ 80, & n_1 = 2 \text{ and } 0 \le n_2 \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that the considered system with impulse response $h(n_1,n_2)$ is separable.

(b) Show that the considered subimage $x(n_1,n_2)$ is separable.

(c) Let $y(n_1,n_2)$ be the output generated by the system when $x(n_1,n_2)$ is applied as the system input. Show how $y(n_1,n_2)$ can be computed by using only two 1-D (one-dimensional) convolutions.

(d) Compute and plot $y(n_1,n_2)$, the output of the system when $x(n_1,n_2)$ is applied as input.

15. **EEE 557–Broadband Networks**

Consider two links in tandem. Packets arrive at a rate λ. Each link is an M/M/1 queue, with independent and exponentially distributed service times of means $1/\mu_1$ and $1/\mu_2$. At the receive end of each link, a fraction and of the packets transmitted are found to be in error. The source (at the transmitter of the first link) is then informed, and the packet has to be retransmitted from the source. Assume that packet transmission times and retransmission times are independent.

(a) Draw the open cyclic queuing network.

(b) Find the average number of packets at the two queues.

(c) Find the mean time it takes before a packet is received correctly.

16. **EEE 505–Time-Frequency Signal Processing**
The short-time Fourier transform (STFT) linear time-frequency representation (TFR) of a signal $x(t)$ is defined as:

$$S_x(t,f) = \int_\tau x(\tau)\ h^*(\tau - t)\ e^{-j2\pi\tau f}\, d\tau,$$

where $h(t)$ is the analysis window. The spectrogram quadratic TFR, $C_x(t,f) = |S_x(t,f)|^2$, is the magnitude squared of the STFT.

(a) Compute the spectrogram of the impulse function $x(t) = \delta(t - t_1)$ using a rectangular window $h(t) = u(t) - u(t-T)$. Discuss what happens to the time resolution of the TFR as T increases.

(b) Provide an alternative expression for the STFT in terms of the Fourier transforms $X(f)$ and $H(f)$ of $x(t)$ and $h(t)$, respectively.

(c) Compute the spectrogram of the sinusoid $x(t) = e^{-j2\pi f_1 t}$ using $H(f) = u(f) - u(f - F)$. Here, $u(f) = \begin{cases} 1, & f > 0 \\ 0, & f < 0 \end{cases}$ is the unit step defined in the frequency domain. Discuss what happens to the frequency resolution of the TFR as F increases.