
ANALOG MODULATION

The sources may be either analog where the messages produced by the source are continuous in time and in amplitude or digital that is discrete in time and has a finite number of possible outputs.

We are now concerned with transmission of analog signals. The source output (i.e., the message signal) is denoted by $m(t)$. We assume that the analog signal, $m(t)$, is lowpass, and its bandwidth is W Hz. The message signal is modulated with a carrier signal of the form

$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

where A_c is the carrier amplitude, f_c is the carrier frequency, and ϕ_c is the carrier phase.

Modulation converts the message signal $m(t)$ from lowpass to bandpass, around the center frequency f_c . The objective of using modulation is to achieve one or more of the following;

- (1) to make the spectrum of the transmitted bandpass signal match the passband characteristics of the channel because the channel may not be suitable for transmitting the source output directly.
- (2) to accommodate simultaneous transmission of different messages over the same channel.
- (3) to provide better noise immunity over a noisy channel by expanding the bandwidth of the transmitted signal.

The third objective is met by employing angle modulation.

In the following labs, we consider the transmission and reception of analog signals by amplitude modulation (AM), frequency modulation (FM) and phase modulation (PM).

LAB 1. AMPLITUDE MODULATION (AM)

Objective

In this part, our objective is to study AMPLITUDE MODULATION in detail. We will consider different methods of amplitude modulation: double-sideband suppressed carrier (DSB-SC) AM, conventional double-sideband AM, and single-sideband AM.

Introduction

In amplitude modulation (AM), the message signal $m(t)$ is impressed on the amplitude of the carrier signal $c(t)$ using several different methods resulting in double-sideband suppressed carrier AM, conventional double-sideband AM, and single-sideband AM.

1-1 DSB-SC AMPLITUDE MODULATION

A general sinusoidal carrier can be represented mathematically as

$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

where f_c is known as the carrier frequency. Since a sinusoid is completely specified by its amplitude, phase, and frequency, it follows that once the carrier frequency is specified, only

two parameters are to be varied in the modulation process: the amplitude A_c and the phase \mathbf{f}_c .

1. Double Sideband Suppressed Carrier AM (DSB-SC AM)

Notation:

$m(t)$: Message signal.

$c(t) = A_c \cos(2\mathbf{p}f_c t + \mathbf{f}_c)$: Carrier signal

$u(t) = A_c m(t) \cos(2\mathbf{p}f_c t + \mathbf{f}_c)$: Modulated signal

*) Modulated signal:

$$u(t) = m(t) \cdot c(t)$$

$$u(t) = A_c m(t) \cos(2\mathbf{p}f_c t + \mathbf{f}_c)$$

Then,

$$U(f) = M(f) * C(f)$$

$$= M(f) * F.T.[A_c \cos(2\mathbf{p}f_c t + \mathbf{f}_c)]$$

By Fourier-transforming the spectrum of modulated signal,

$$U(f) = M(f) * C(f)$$

$$= M(f) * \left\{ \frac{A_c}{2} e^{j\mathbf{f}_c} \mathbf{d}(f - f_c) + \frac{A_c}{2} e^{-j\mathbf{f}_c} \mathbf{d}(f + f_c) \right\}$$

$$\therefore U(f) = \frac{A_c}{2} e^{j\mathbf{f}_c} M(f - f_c) + \frac{A_c}{2} e^{-j\mathbf{f}_c} M(f + f_c)$$

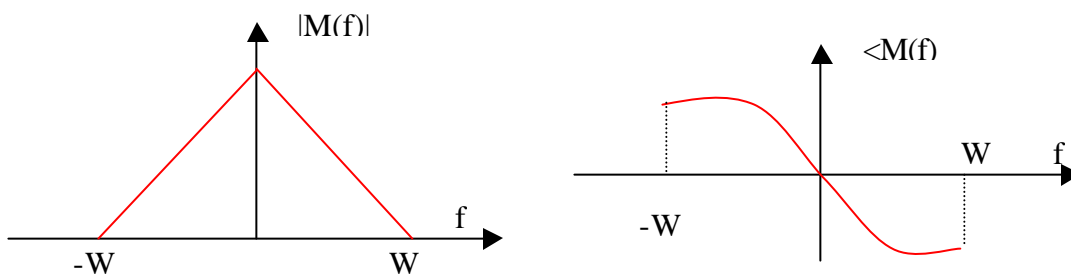
If we assume that the phase of carrier signal is zero, we obtain:

$$U(f) = \frac{A_c}{2} M(f - f_c) + \frac{A_c}{2} M(f + f_c)$$

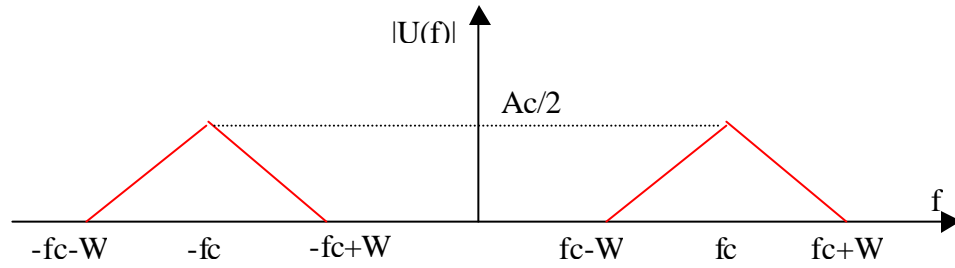
Thus, the required bandwidth of the modulated signal is equal to twice the bandwidth of the message signal as it is clear from the expression.

EXAMPLE :

Assume $m(t)$ is such that:

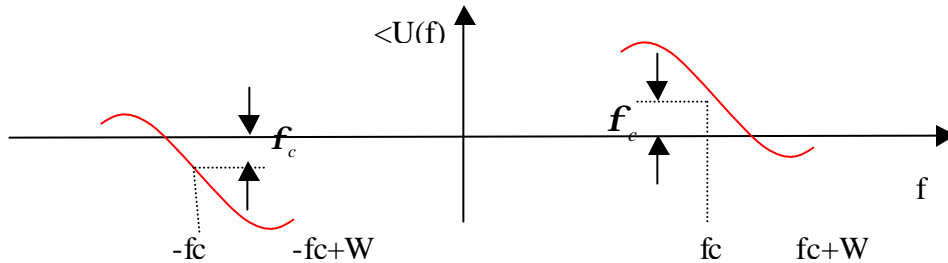


$$\Rightarrow |U(f)| = \frac{A_c}{2} |M(f - f_c)| + \frac{A_c}{2} |M(f + f_c)| \text{ since } f_c > 2W$$



$$\Rightarrow \angle U(f) = \angle M(f - f_c) + f_c, \text{ if } f > 0$$

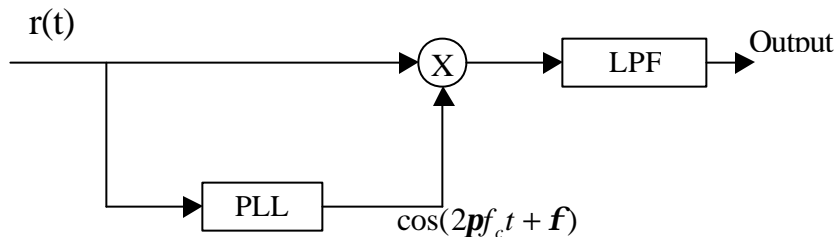
$$= \angle M(f + f_c) - f_c, \text{ if } f < 0$$



If channel attenuation and noise are ignored, the received signal can be represented as follows

$$r(t) = u(t)$$

The receiver is composed of a PLL (Phase-Locked Loop) and a LPF (Lowpass Filter).



$$r(t) \cos(2pf_c t + f) = u(t) \cos(2pf_c t + f)$$

$$= A_c \cdot m(t) \left[\frac{1}{2} \cos(f_c - f) + \frac{1}{2} \cos(4pf_c t + f + f_c) \right]$$

At the output of LPF:

$$\frac{1}{2} A_c \cdot m(t) \cos(f_c - f)$$

When we assume that PLL works perfectly, the output of the receiver would be

$$\frac{1}{2} A_c \cdot m(t).$$

Therefore the message signal is demodulated.

1-2 CONVENTIONAL AMPLITUDE MODULATION

A conventional AM signal consists of a large carrier component in addition to the DSB AM modulated signal. The transmitted signal is expressed mathematically as

$$\begin{aligned} u(t) &= A_c [1 + m(t)] \cos(2\pi f_c t + \mathbf{f}_c) \\ &= A_c (1 + a \cdot m_n(t)) \cos(2\pi f_c t + \mathbf{f}_c) \end{aligned}$$

where $|m(t)| \leq 1$,

a : modulation index. $a \leq 1$ is chosen.

If $a > 1$, overmodulated. $a = \max |m(t)|$.

Similar to the DSB-SC AM case, the spectrum of a conventional AM signal occupies a bandwidth twice of the bandwidth of the message signal.

We can demodulate the modulating signal simply at the receiver by using an envelope detector. Envelope detector consists of a diode and RC circuit. It is essentially a lowpass filter which charges a capacitor to the peak amplitude of the incoming AM modulated signal as long as the output of the envelope detector is larger than the input of the envelope detector. When the amplitude of the incoming AM signal decreases, the capacitor discharges. The major advantage of an envelope detector is very simple, therefore cheap.

There is another demodulation method that recovers the carrier signal from the received signal and uses coherent demodulation. The modulated signal can be represented as follows

$$u(t) = A_c m(t) \cos(2\pi f_c t + \mathbf{f}_c) + A_c \cos(2\pi f_c t + \mathbf{f}_c).$$

When a narrow-bandpass filter that is centered at f_c is used at the receiver, the carrier signal can be recovered (i.e., $A_c \cos(2\pi f_c t + \mathbf{f}_c)$ can be obtained from the received signal).

We then multiply the received signal with the carrier signal and lowpass filter to demodulate the message signal (similar to the case of DSB-SC AM).

1-3 SINGLE SIDEBAND AMPLITUDE MODULATION

Single sideband amplitude modulation scheme is similar to the double sideband-suppressed carrier AM scheme. The spectrum of DSB-SC AM modulated signal has the scaled conjugate symmetric spectrums of the message signal on both positive and negative frequency ranges. The idea of single sideband AM is that only one side spectrums on either the upper or the lower sideband from the carrier signal can be used to demodulate the message signal. Only the difference between two schemes is SSB (Single sideband) AM modulated signal requires half the bandwidth of DSB-SC AM modulated signal. Therefore it increases the bandwidth efficiency compared to DSB-SC AM on the bandwidth limited channel.

There exist two types of SSB-AM, USB-AM and LSB-AM, depending on either the upper or the lower sideband. The modulated signal is represented by

$$u(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) \mp \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

where the minus sign determines USB-AM and the plus sign determines LSB-AM. $\hat{m}(t)$ is the Hilbert transform of $m(t)$, denoted by $\hat{m}(t) = \frac{1}{\pi t} * m(t)$.

In the frequency domain we have

$$U_{USB}(f) = \begin{cases} M(f - f_c) + M(f + f_c), & f_c \leq |f| \\ = 0 & \text{otherwise} \end{cases}$$

and

$$U_{LSB}(f) = \begin{cases} M(f - f_c) + M(f + f_c), & f_c \geq |f| \\ = 0 & \text{otherwise.} \end{cases}$$

The bandwidth of the SSB signal is half the bandwidth of the DSB and conventional AM and it is equal to the bandwidth of the message signal.

The demodulation is done in exactly the same way as DSB-SC AM, i.e., by a coherent demodulator.

Problem 1-1. DSB-SC AM with a rectangular message signal.

Let $m(t)$ be a rectangular signal having an amplitude of 1 and having a pulse-duration of 50ms. The carrier signal frequency is 3kHz. Assume that the phase of the carrier signal is zero and the amplitude of the carrier signal is 1.

[Hint] In **Analog Comm.** on the drop-down menu, **Cont. Sig. Gen.** block is followed by modulation blocks.

- Show the spectrum of the message signal, $M(f)$.
- Show the spectrum of the DSB-SC AM modulated signal, $U(f)$.

Problem 1-2. DSB-SC AM with a sinc function as a message signal.

Let $m(t) = \text{sinc}(200t)$ for $|t| < 50\text{ms}$
 $= 0$ for otherwise.

Assume that the phase of carrier signal is zero and the amplitude of the carrier signal is 1, and the carrier signal frequency is 1 kHz.

- Show the spectrum of the message signal, $M(f)$.
- Show the DSB-SC AM modulated signal both in time domain and in frequency domain.

Problem 1-3. DSB-SC AM with a sinusoidal message signal having a single frequency component.

Let $m(t) = \sin(2\pi f_m t)$ for $0 \leq t \leq 10ms$ and 0 for otherwise, where f_m is 1kHz.

Carrier frequency is 10kHz.

Assume that the phase of carrier signal is zero and the amplitude of the carrier signal is 1.

- Show the spectrum of the message signal, $M(f)$.
- Show the DSB-SC AM modulated signal both in time domain and in frequency domain.

Problem 1-4. Single sideband AM with a sinusoidal message signal.

Let $m(t) = 2\sin(2\pi f_m t)$ for $0 \leq t \leq 10ms$ and 0 for otherwise, where f_m is 200Hz.

Carrier frequency is 3kHz.

Assume that the phase of carrier signal is zero and the amplitude of the carrier signal is 1.

- Show the spectrum of the message signal, $M(f)$.
- Show the USB-AM modulated signal both in time domain and in frequency domain.
- Show the LSB-AM modulated signal both in time domain and in frequency domain.

Problem 1-5. Conventional AM with a sinusoidal message signal.

Let $m(t) = \frac{1}{2} \sin(2\pi f_m t)$ for $0 \leq t \leq 10ms$ and 0 for otherwise, where f_m is 400Hz.

Carrier frequency is 5kHz.

Assume that the phase of carrier signal is zero and the amplitude of the carrier signal is 1.

- Find the modulation index, a , mathematically.
- Show the conventional AM modulated signal both in time domain and in frequency domain.

Problem 1-6. Over modulated signal.

Let $m(t) = A \cdot \sin(2\pi f_m t)$ for $0 \leq t \leq 10ms$ and 0 for otherwise, where f_m is 200Hz.

Carrier frequency is 10kHz.

Assume that the phase of carrier signal is zero and the amplitude of the carrier signal is 1.

- Find the modulation index, a , mathematically.

- b) When A is equal to 0.5, show the modulated signal in time domain with your modulation index used.
- c) When A is equal to 2, show the modulated signal in time domain with your modulation index used.
- d) Explain the effect of overmodulation.

Problem 1-7. Demodulation of DSB-SC AM modulated signal.

Let us consider signals in Problem 1-2.

Assume that there is no noise.

[Hint] After connecting **Channel** block at the end of modulation block, **Channel** block is followed by **Coherent Detection** block. And **Demod.** block is connected at the end of **Coherent Detection** block.

- a) Show the demodulated message signal, $m'(t)$ with your specific bandwidth of LPF.
- b) Show both the spectrum of the demodulated message signal, $M'(f)$, and the demodulated message signal, $m'(t)$ at several bandwidths of LPF, such as $W/4, W, \text{ and } 2W$, where W is the bandwidth of the message signal.
- c) Find the minimum required bandwidth of LPF for reasonable resolution of the demodulated signal.

Problem 1-8. Demodulation of DSB-SC AM modulated signal (2).

Let us consider signals in Problem 1-3.

Assume that there is no noise.

- a) Show the demodulated message signal, $m'(t)$.
- b) Show the spectrum of the demodulated message signal, $M'(f)$ at several bandwidths of LPF, such as $f_m/2, f_m, 2f_m$ and $5f_m$.
- c) Find the minimum required bandwidth of LPF for reasonable resolution of the demodulated signal.

Problem 1-9. Demodulation of SSB-AM modulated signal.

Let us consider signals in Problem 1-4.

Assume that there is no noise.

[Hint] After connecting **Channel** block at the end of modulation block, **Channel** block is followed by **Coherent Detection** block. And **Demod.** block is connected at the end of **Coherent Detection** block.

- a) Show the demodulated message signal, $m'(t)$ with your specific bandwidth of LPF. Assumed that USB is used.

- b) Show both the spectrum of the demodulated message signal, $M'(f)$, and the demodulated signal, $m'(t)$ at several bandwidths of LPF, such as f_m , $2f_m$ and $5f_m$.
- c) Find the minimum required bandwidth of LPF for reasonable resolution of the demodulated signal.

Problem 1-10. Envelope Detector for Conventional AM modulated signal.

Let us consider signals in Problem 1-6.

Let us replace the time duration with $0 \leq t \leq 50ms$ and $\max|m(t)|$ is equal to 1.

Assume that there is no noise.

[Hint] After connecting **Channel** block at the end of modulation block, **Channel** block is followed by **Envelope Detection** block.

- a) Show both the demodulated message signal, $m'(t)$ and the spectrum of the demodulated signal, $M'(f)$ at several time constant values between $1/f_m$ and $1/f_c$.
- b) Explain the relationship between the resolution of the demodulated signal and a time constant.

Quiz.

1. Let the DSB-SC AM modulated signal be $u(t)$ and let the message signal be $m(t)$. Show the relationship between $U(f)$ and $M(f)$ mathematically.
2. In DSB-SC AM, assume the modulating signal, $m(t)$, is a sinusoidal signal. Show the plots of $U(f)$ on the cases of both $f_m < f_c$ and $f_m > f_c$ with J-DSP and explain the results.
3. How can we recover $m(t)$ from the DSB-SC AM modulated signal, $u(t)$?
And determine the demodulated signal.
Assumed that the phase of the carrier at the receiver is known as \hat{f} by PLL and $r(t)$ is equal to $u(t)$ in the absence of noise.
Show a block diagram if necessary.
4. What is the bandwidth of the DSB-SC AM modulated signal? Assumed W is equal to the bandwidth of the message signal.
 - 1) $W/2$
 - 2) W
 - 3) $3W/2$
 - 4) $2W$
5. When we look at the spectral characteristics of the conventional AM modulated signal, explain the difference of the conventional AM with DSB-SC AM.
6. How can we recover $m(t)$ from the conventional AM modulated signal, $u(t)$?

Assumed that envelope detector is used to demodulate the message signal in the absence of noise.

7. How can we recover $m(t)$ from the conventional AM modulated signal, $u(t)$, with a pilot tone?
Assumed that noise is absence in channel.
Show a block diagram if necessary.
8. What is the bandwidth of the conventional AM modulated signal? Assumed W is equal to the bandwidth of the message signal.
 - 1) $W/2$
 - 2) W
 - 3) $3W/2$
 - 4) $2W$
9. Let the USSB-AM modulated signal be $u(t)$ and let the message signal be $m(t)$.
Show the relationship between $U(f)$ and $M(f)$ mathematically.
10. What is the bandwidth of the SSB-AM modulated signal? Assumed W is equal to the bandwidth of the message signal.
 - 1) $W/2$
 - 2) W
 - 3) $3W/2$
 - 4) $2W$