(5.9)

Find the Laplace transforms of the following time waveforms

(a) $e^{-2t}[u(t)-u(t-1)]$

(b) $(t+1)\,u(t-1)$

(c) $e^{-2t}\,u(t-2)$

(d) $\frac{d}{dt}\left[e^{-t}\sin 2t\right]$

(a) $e^{-2t}u(t) - e^{-2(t'+1)}u(t') = e^{-2t}(1-e^{-2})u(t)$

$$\mathcal{L}\{e^{-2t}[u(t)-u(t-1)]\} =$$

$$= \int_0^\infty e^{-st}e^{-2t}[u(t)-u(t-1)]\,dt =$$

$$= \int_0^1 e^{-(2+s)t}\,dt = -\frac{1}{2+s}\,e^{-(2+s)t}\Big|_0^1$$

$$= -\frac{1}{2+s}\left[e^{-2+s}-1\right] = \frac{1}{s+2}\left[1-e^{-2+s}\right]$$

(b) $$\mathcal{L} = \int_0^\infty (t+1)\,u(t-1)\,e^{-st}\,dt = \int_1^\infty (t+1)e^{-st}\,dt =$$

$$= \int_1^\infty t e^{-st}\,dt + \left(-\frac{1}{s}\right)\Big/\, e^{-st} = -\frac{1}{s}\int_1 t\,d(e^{-st}) + e^{-s}$$

$$= \left(-\frac{t}{s}\right)e^{-st}\Big|_1^\infty + \frac{1}{s}\int_1^\infty e^{-st}\,dt = \frac{1}{s}e^{-s} + e^{-s} - \frac{1}{s^2}\,e^{-st}\Big|_1^\infty$$

$$= \left(\frac{1}{s^2}+\frac{1}{s}+1\right)e^{-st}$$

(c) $\mathcal{L}\left(e^{-2t}u(t-2)\right) = \int_0^\infty e^{-st}e^{-2t}\,u(t-2)\,dt$

$$= \int_0^\infty e^{-(s+2)t}\,u(t-2)\,dt = \int_2^\infty e^{-(s+2)t}\,dt$$

$$= -\frac{1}{s+2}\,e^{-(s+2)t}\Big|_2^\infty = \frac{1}{s+2}\,e^{-2(s+2)}$$

(d) $\mathcal{L}\left[\frac{d}{dt}e^{-t}\sin 2t\right] = \int_0^\infty e^{-st}\frac{d}{dt}\left(e^{-t}\sin 2t\right)dt$

$$= e^{-st}e^{-t}\sin 2t\Big|_0^\infty + s\int_0^\infty e^{-(s+1)t}\sin 2t\,dt$$

$$= s\int_0^\infty e^{-(s+1)t}\,\frac{e^{j2t}-e^{-2jt}}{2j}\,dt =$$

$$= \frac{s}{2j}\left[\frac{e^{-[(s+1)-j2]t}}{-(s+1)+j2} - \frac{e^{-(s+1+j2)t}}{-(s+1)-j2}\right]\Bigg|_0^\infty$$

$$= \frac{s}{2j}\,\frac{\cancel{-(s+1)}+j2+\cancel{(s+1)}-j2}{-(s+1)^2-2^2} = \frac{2s}{(s+1)^2+2^2}$$

(5.10) Find the Laplace transforms of the following time waveforms.

(a) $x_a(t) = \begin{cases} 1 & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(s) = (1-e^{-sT})\,\frac{1}{s}$

(b) $x_b(t) = t^2e^{-3t},\ t>0 \quad \leftrightarrow \quad \frac{1}{(s+3)^3}$

(c) $x_c(t) = e^{-4t}\sin 5t,\ t>0 \quad \leftrightarrow \quad X(s) = \frac{5}{(s+4)^2+25}$

(d) $x_d(t) = t,\ t>0 \quad \leftrightarrow \quad X(s) = \frac{1}{s^2}$

(5.12) Find the Laplace transforms of the following signals:

(a) $x_a(t) = \begin{cases} 3 & t \geq 0 \\ 1 & t < 0 \end{cases}$ (b) $x_b(t) = \begin{cases} 10e^{-2t}, & t \geq 3 \\ 0, & t < 3 \end{cases}$

(a) $\mathcal{L}(x_a(t)) = \int_0^\infty 3e^{-st}\,dt = -\frac{3}{s}e^{-st}\Big|_0^\infty = \frac{3}{s}$

(b) $\mathcal{L}(x_b(t)) = \int_0^\infty 10e^{-2t}u(t-3)\,dt\, e^{-st} =$

$= 10\int_3^\infty e^{-(2+s)t}\,dt = -\frac{10}{2+s}e^{-(2+s)t}\Big|_3^\infty =$

$= \frac{10}{2+s}e^{-(2+s)\cdot 3}$

(5.13) Construct the inverse Laplace transform of

$X(s) = \frac{s+a-a}{(s+a)^2+b^2} = \frac{s+a}{(s+a)^2+b^2} - \frac{a}{(s+a)^2+b^2}$

$x(t) = e^{-at}\cos(bt) - \frac{a}{b}e^{-at}\sin(bt)$

(5.16) Construct the inverse laplace transforms of the following functions:

$X_a(s) = \frac{5s}{2s+1}$ $X_b(s) = \frac{2s-1}{(s+3)^2+4}$ $X_c(s) = \frac{7s^2}{s^3+5s^2+1}$

(a) $\mathcal{L}^{-1}\{X_a(s)\} \Rightarrow \frac{5s}{2s+1} = \frac{5}{2}\frac{s}{s+1/2} = \frac{5}{2}\frac{s+1/2-1/2}{s+1/2} =$

$= \frac{5}{2}\left\{1 - \frac{1}{2}\frac{1}{s+1/2}\right\} \longrightarrow \frac{5}{2}\left[\delta(t) - \frac{1}{2}e^{-t/2}u(t)\right]$

(b) $\frac{2s-1}{(s+3)^2+4} = 2\frac{s-1/2}{(s+3)^2+4} = 2\frac{s+3}{(s+3)^2+4} - 2\frac{3+1/2}{(s+3)^2+4}$

$\mathcal{L}^{-1}(b) = 2e^{-3t}\cos 2t - \frac{7}{2}e^{-3t}\sin 2t$

(5-19) Construct the inverse Laplace transforms of the following functions

$$Y_a(s) = \frac{3}{2s^2+1}$$

$$Y_b(s) = \frac{s-3}{s^2+6s+9}$$

$$Y_c = \frac{6s}{(s^2+1)(s^2+4)}$$

(a) $Y_a(s) = \frac{3}{2s^2+1} = \frac{3}{2}\,\frac{1/\sqrt{2}}{s^2+1/2}\cdot\sqrt{2} \Rightarrow \frac{3\sqrt{2}}{2}\sin\left(\frac{1}{\sqrt{2}}t\right) = Y_a(t)$

(b) $Y_b(s) = \frac{s-3}{s^2+6s+9} = \frac{s-3}{(s+3)^2} = \frac{A}{s+3} + \frac{B}{(s+3)^2}$

$\frac{s-3}{s^2+6s+9} = \frac{A}{s+3} + \frac{B}{(s+3)^2} \quad / \cdot (s+3)^2$

$s-3 = A(s+3) + B$

$s = -3 \quad B = -6$

$1 = A \quad A = 1$

$\frac{s-3}{s^2+6s+9} = \frac{1}{s+3} - \frac{6}{(s+3)^2}$

$\mathcal{L}^{-1}() = e^{-3t} - 6te^{-3t}$

(c) $Y_c = \frac{6s}{(s^2+1)(s^2+4)} = \frac{A}{s^2+1} + \frac{B}{s^2+4}$

$\frac{6s}{s^2+4} = A + \frac{B(s^2+1)}{s^2+4} \qquad s^2 = -1 = i^2 \qquad A = \frac{6i}{4-1} = 2i$

$s = i$

$\frac{6s}{s^2+1} = A\frac{s^2+4}{s^2+1} + B \qquad s^2 = -4 \qquad B = \frac{6\cdot i2}{1-4} = -4i$

$s = i2$

$Y_c = 2i\frac{1}{s^2+1} - 4i\frac{1}{s^2+4} = 2i\frac{1}{s^2+1} - 2i\frac{2}{s^2+4}$

$Y_c(t) = 2i\sin(t) - 2i\sin(2t) = 2i(\sin t - \sin 2t)$