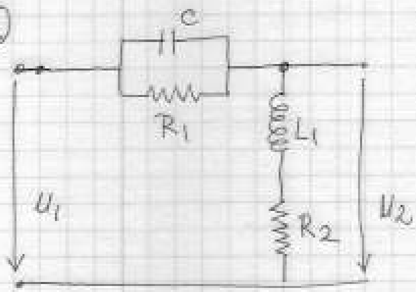
Solutions to HWK #8 EEE 202 Spring 2007

(1)



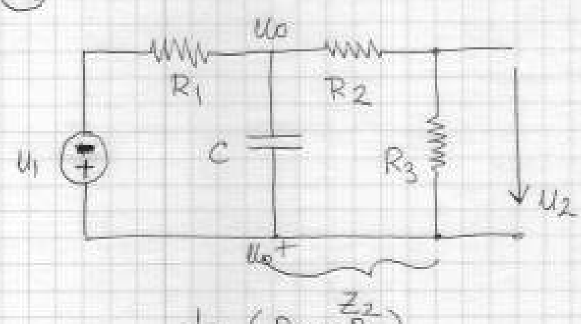
$R_1 = 1K,\ R_2 = 2K,\ L_1 = 0.1H$
$C_1 = 10^{-6}$

$$\frac{u_2(s)}{u_1(s)} = \frac{R_2 + sL_1}{R_2 + sL_1 + \dfrac{R_1 \, 1/sC}{R_1 + \frac{1}{sC}}}$$

$$= \frac{R_2 + sL_1}{R_2 + sL_1 + \frac{R_1}{1 + sCR_1}} =$$

$$= \frac{(R_2 + sL_1)(1 + sCR_1)}{R_1 + (R_2 + sL_1)(1 + sCR_1)}$$

(2.) $u_1(t) = u_0 + U_{m1}\sin(\omega_0 t + \varphi_1) + U_{m3}\sin(3\omega_0 + \varphi_3)$



$R_1 = 1k\Omega,\ R_2 = 2k\Omega$
$R_3 = 4k\Omega,\ C = 1\mu F$
$U_0 = 5V,\ U_{m1} = 10V,\ U_{m3} = 2V$
$\omega_0 = 1000\ rad/s,\ \varphi_1 = \pi/4,\ \varphi_3 = -\pi/3$

$$Z_2 = \frac{\frac{1}{j\omega C}(R_2 + R_3)}{\frac{1}{j\omega C} + (R_2 + R_3)} = \frac{R_2 + R_3}{1 + j\omega C(R_2 + R_3)}$$

$$u_0 = \frac{Z_2 u_1}{R_1 + Z_2} = \frac{\frac{R_2 + R_3}{1 + j\omega C(R_2 + R_3)}}{R_1 + \frac{R_2 + R_3}{1 + j\omega C(R_2 + R_3)}} u_1 = \frac{(R_2 + R_3)\, u_1}{R_1(1 + j\omega C(R_2 + R_3)) + R_2 + R_3}$$

$$= \frac{R_2 + R_3}{R_1 + R_2 + R_3 + j\omega C(R_2 + R_3)R_1} u_1$$

$$u_2 = \frac{R_3}{R_2 + R_3} u_0 = \frac{R_3\, u_1}{R_1 + R_2 + R_3 + j\omega C(R_2 + R_3)R_1}$$

$$\underline{u}_2 = \frac{R_3 \, \underline{u}_1}{R_1 + R_2 + R_3 + j\omega C R_1 (R_2 + R_3)}$$

$$\underline{u}_2 = \frac{4K \, \underline{u}_1}{1K + 2K + 4K + jn1\cancel{K} \cdot 1K \cdot 1\mu (2K + 4K)} =$$

$$= \frac{4K \, \underline{u}_1}{7K + jn6K} = \frac{4}{7 + j6n} \underline{u}_1$$

1) $u_1 = u_0, \ \omega = 0, \ n = 0 \qquad u_2 = \frac{4}{7} \cdot 5 = \frac{20}{7}$

2) $u_1 = U_{m1}, \ \omega = \omega_0, \ n = 1 \qquad \underline{u}_2 = \frac{4}{7 + j6} 10 \angle \pi/4 =$

$$= \frac{40}{\sqrt{49 + 36}} \angle \pi/4 - \tan^{-1}\left(\frac{6}{7}\right)$$

$$= \frac{40}{\sqrt{85}} \angle \pi/4 - \tan^{-1}\left(\frac{6}{7}\right) = u_{02} \angle \theta_2$$

3) $u_3 = u_{m3}, \ \omega = 3\omega_0, \ n = 3 \qquad \underline{u}_3 = \frac{4}{7 + j18} 2 \angle -\pi/3$

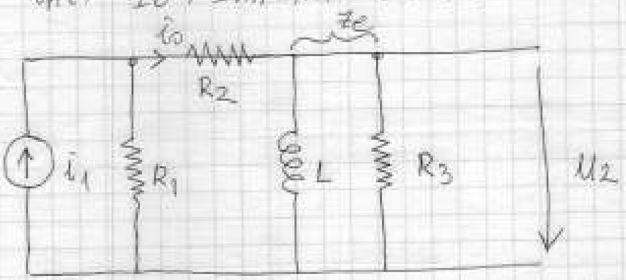
$$\underline{u}_3 = \frac{8}{\sqrt{49 + 18^2}} \angle -\pi/3 - \tan^{-1}\left(\frac{18}{7}\right) = u_{03} \angle \theta_3$$

$$u_2(t) = \frac{20}{7} + \frac{40}{\sqrt{85}} \sin\left(\omega_0 t + \pi/4 - \tan^{-1}(6/7)\right)$$

$$+ \frac{8}{\sqrt{49 + 18^2}} \sin\left(3\omega_0 t - \pi/3 - \tan^{-1}\left(\frac{18}{7}\right)\right)$$

$$u_{2rms} = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{20}{7}\right)^2 + \frac{40^2}{85} + \frac{64}{49 + 18^2}}$$

③ $i_1(t) = I_0 + I_{m1}\sin(\omega_0 t + \varphi_1) + I_{m3}\sin(3\omega_0 t + \varphi_3)$



$R_1 = 1K,\ R_2 = 2K,\ R_3 = 4K,\ L = 1H$

$I_0 = 5mA,\ I_{m1} = 10mA,\ I_{m3} = 2mA$

$\omega_0 = 1000\,rad/s,\ \varphi_1 = \pi/4,\ \varphi_3 = -\pi/3$

$$Z_e = \frac{R_3\ j\omega L}{R_3 + j\omega L} = \frac{j\omega L R_3}{R_3 + j\omega L}$$

$$I_0 = \frac{R_1}{R_1 + R_2 + \frac{j\omega L R_3}{R_3 + j\omega L}} = \frac{R_1(R_3 + j\omega L)}{(R_1 + R_2)(R_3 + j\omega L) + j\omega L R_3} I_1$$

$$U_2 = \frac{j\omega L R_3}{\cancel{R_3 + j\omega L}} \cdot \frac{R_1(\cancel{R_3 + j\omega L})}{(R_1 + R_2)(R_3 + j\omega L) + j\omega L R_3} I_1$$

$$U_2 = \frac{j\omega L R_1 R_3}{(R_1 + R_2)(R_3 + j\omega L) + j\omega L R_3} I_1$$

$$U_2 = \frac{j1K \cdot 1K \cdot 4Kn}{3K(4K + jKn)} I_1 = \frac{j\ 4Kn}{3(4 + jn)} I_1$$

1) $I_1 = I_0 = 5mA$, $\omega = 0$, $n = 0$ $\quad U_2 = 0$

2) $I_2 = I_{m1} = 10mA \angle \pi/4 \qquad n = 1$

$$U_{22} = \frac{j4\cancel{k} \cdot 10m\angle \pi/4}{3(4+j)} = \frac{40 \angle \pi/4 + \pi/2}{3\sqrt{17} \angle \tan^{-1}(1/4)}$$

$$U_{22} = \frac{40}{3\sqrt{17}} \angle \pi/4 + \pi/2 - \tan^{-1}(1/4)$$

3) $I_3 = I_{m3} = 2mA \angle -\pi/3$

$$U_{23} = \frac{j4 \times 3 \times 2 \angle -\pi/3}{3(4+j3)} = \frac{\cancel{24}\,8}{\cancel{3}\sqrt{16+9}} \angle \frac{\pi}{2} - \frac{\pi}{3} - \tan^{-1}\left(\frac{3}{4}\right)$$

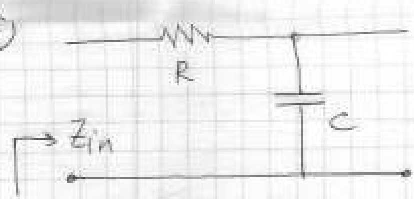
$$U_{23} = \frac{8}{5} \angle \frac{\pi}{2} - \frac{\pi}{3} - \tan^{-1}\left(\frac{3}{4}\right)$$

$$U_2(t) = \frac{40}{3\sqrt{17}} \sin\left(\omega_0 t + \frac{\pi}{4} + \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)\right)$$

$$+ \frac{8}{5} \sin\left(3\omega_0 + \frac{\pi}{2} - \frac{\pi}{3} - \tan^{-1}\left(\frac{3}{4}\right)\right) \quad \checkmark$$

$$U_{2rms} = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{40}{3\sqrt{17}}\right)^2 + \left(\frac{8}{5}\right)^2}$$

④



1) Impedance

$$\underline{Z} = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C}$$

2) $v(t) = v\, e^{j\omega t}$ $\quad \underline{V} = v\angle 0$

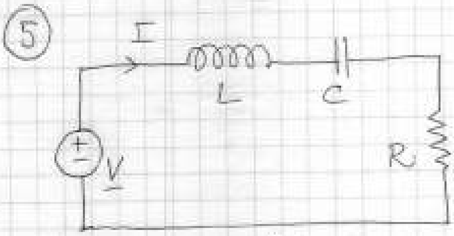
$$I_c = \frac{V}{Z} = \frac{v\angle 0}{\sqrt{R+\left(\frac{1}{\omega C}\right)^2}\ \angle -\tan^{-1}\left(\frac{1}{\omega RC}\right)} = \frac{v}{\sqrt{R+\left(\frac{1}{\omega C}\right)^2}}\ \angle \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

$$i_c = \frac{v}{\sqrt{R+\left(\frac{1}{\omega C}\right)^2}} \cos\left(\omega t + \tan^{-1}\left(\frac{1}{\omega RC}\right)\right)$$

3) $$\underline{V_C} = \frac{1}{j\omega C} I_c = \frac{1}{j\omega C} \frac{v}{\sqrt{R+\left(\frac{1}{\omega C}\right)^2}}\ \angle \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{\omega RC}\right) - \frac{\pi}{2}$$

⑤



$\omega = 1k$ rad/s $\quad R = 1k\Omega,\ L = 1H$
$C = 10\mu F$

(1) $$I = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \qquad |I| = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

(2) $$\varphi_I = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

(3) $$u_L = j\omega L \underline{I} \qquad |u_L| = \frac{\omega L V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

(4) $$\theta_L = \pi/2 - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$ (5) $$\omega = \frac{1}{\sqrt{LC}}$$