CHAPTER 7 PROBLEMS

7.1 Find the frequency domain impedance **Z**, shown in Fig. 7.1.

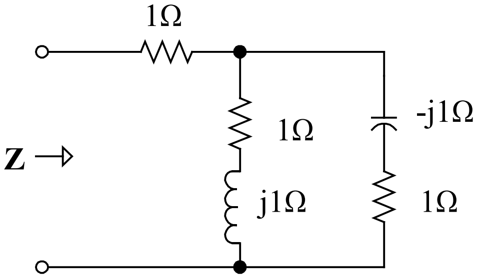

Fig. 7.1

7.2 If the impedance of the network in Fig. 7.2 is real at f = 60Hz, what is the value of the inductor L?

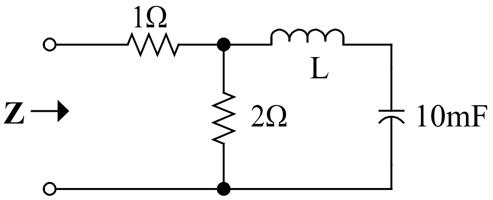

Fig. 7.2

7.3 Use nodal analysis to find $\mathbf{V}_0$ in the network in Fig. 7.3.

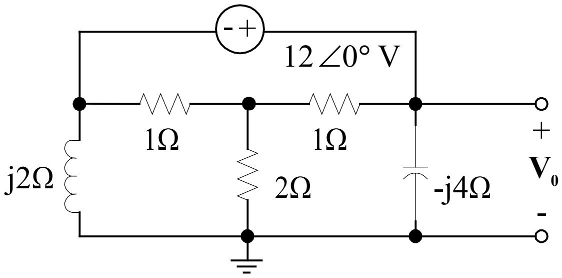

Fig. 7.3

7.4 Find $\mathbf{V}_0$ in the network in Fig. 7.4 using (a) loop analysis (b) superposition and (c) Thevenin's Theorem.

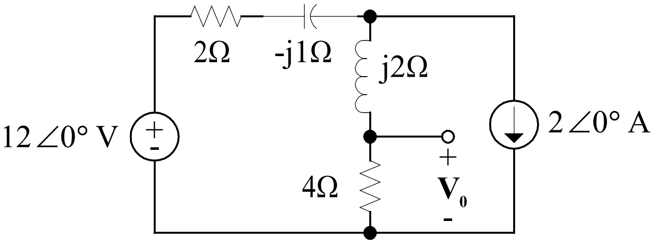

Fig. 7.4

CHAPTER 7 SOLUTIONS

7.1. To begin our analysis, we note that the circuit can be labeled as shown in Fig. S7.1.

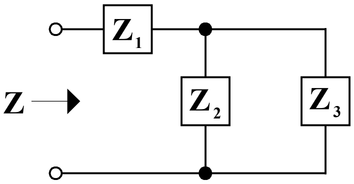

Fig. S7.1

In this case, $\mathbf{Z_1}$ consists of a 1Ω resistor, $\mathbf{Z_2}$ is the series combination of a 1Ω resistor and a j1Ω inductor and $\mathbf{Z_2}$ consists of a –j1Ω capacitor in series with a 1Ω resistor. Therefore,

$$\mathbf{Z_1} = 1\Omega$$
$$\mathbf{Z_2} = 1 + j1\Omega$$
$$\mathbf{Z_3} = 1 - j1\Omega$$

Starting at the opposite end of the network from the terminals at which **Z** is calculated we note that $\mathbf{Z_2}$ and $\mathbf{Z_3}$ are in parallel and their combination is in series with $\mathbf{Z_1}$. Hence

$$\begin{aligned}\mathbf{Z} &= \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 \\ &= 1 + \frac{(1+j)(1-j)}{1+j+1-j} \\ &= 1 + \frac{2}{2} \\ &= 2\Omega\end{aligned}$$

7.2 The general expression for the impedance of this network is

$$\mathbf{Z} = 1 + 2 \left\| \left(j\omega L + \frac{1}{j\omega C} \right) \right.$$

In order for **Z** to be purely resistive, the term $\left(j\omega L + \frac{1}{j\omega C} \right)$ must be real, i.e.

$$\mathbf{Z_{LC}} = R_{LC} + j0$$

However, since $\mathbf{Z_{LC}}$ can be written as

$$\mathbf{Z}_{LC} = j\left(\omega L - \frac{1}{\omega C} \right)$$

it is clearly an imaginary term and $R_{LC} = 0$. Therefore, in order for **Z** to be resistive

$$\omega L - \frac{1}{\omega C} = 0$$

or

$$\begin{aligned} L &= \frac{1}{\omega^2 C} \\ &= \frac{1}{(377)^2 (10^{-2})} \\ &= 703.6\mu H \end{aligned}$$

7.3 The presence of the voltage source indicates that nodal analysis is a viable approach to this problem. The voltage source and its two connecting nodes form a supernode as shown in Fig. S7.3.

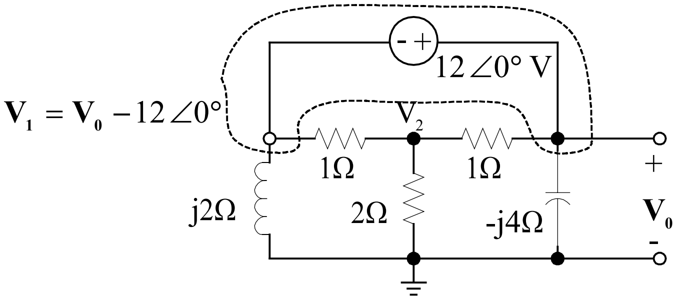

Fig. S7.3

Note that there are three non-reference nodes, i.e., $\mathbf{V_1}$, $\mathbf{V_2}$ and $\mathbf{V}_0$. Because the voltage source is tied directly between nodes $\mathbf{V_1}$ and $\mathbf{V}_0$, $\mathbf{V}_1 = \mathbf{V}_0 - 12\angle 0°$. This constraint condition is one of our three equations required to solve the network. The two remaining equations are obtained by applying KCL at the supernode and the node labeled $\mathbf{V_2}$. For the supernode, KCL yields

$$\frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} + \frac{\mathbf{V}_0 - \mathbf{V}_2}{1} + \frac{\mathbf{V}_0}{-j4} = 0$$

At the node labeled $\mathbf{V_2}$, KCL yields

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{1} + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_0}{1} = 0$$

Therefore, the three equations that will provide the node voltages are

$$\mathbf{V}_1 = \mathbf{V}_0 - 12$$

$$-j\frac{1}{2}\mathbf{V}_1 + \mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_0 - \mathbf{V}_2 + j\frac{1}{4}\mathbf{V}_0 = 0$$

$$\mathbf{V}_2 - \mathbf{V}_1 + \frac{1}{2}\mathbf{V}_2 + \mathbf{V}_2 - \mathbf{V}_0 = 0$$

Substituting the first equation in for the two remaining equations and combining terms yields

$$\mathbf{V}_0\left(2 - j\frac{1}{4}\right) - 2\mathbf{V}_2 = 12 - j6$$

$$-2\mathbf{V}_0 + \frac{5}{2}\mathbf{V}_2 = -12$$

Solving for $\mathbf{V_2}$ in this last equation and substituting it into the one above it, we obtain

$$\mathbf{V}_0\left(0.4 - j0.25\right) = 2.4 - j6$$

and hence

$$\mathbf{V}_0 = 13.57 \angle -36.2° \text{ V}$$

7.4 (a) Since the network has two loops, or in this case two meshes, we will need two equations to determine all the currents. Consider the network as labeled in Fig. S7.4(a).

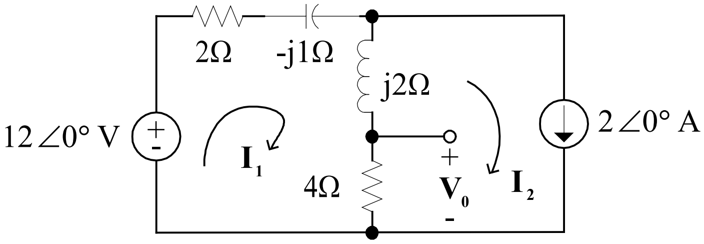

Fig. S7.4(a)

Note that since $\mathbf{I_2}$ goes directly through the current source, $\mathbf{I_2}$ must be 2∠0°A. Hence, one of our two equations is

$$\mathbf{I}_2 = 2\angle 0°$$

If we now apply KVL to the loop on the left of the network, we obtain

$$-12 + \mathbf{I}_1\left(2 - j1\right) + \left(\mathbf{I}_1 - \mathbf{I}_2\right)\left(4 + j2\right) = 0$$

These two equations will yield the currents. Substituting the first equation into the second yields

$$-12 + \mathbf{I}_1 (2 - \mathrm{j}1 + 4 + \mathrm{j}2) - 2(4 + \mathrm{j}2) = 0$$

and then

$$\mathbf{I}_1 = \frac{20 + \mathrm{j}4}{6 + \mathrm{j}1} = 3.35\angle 1.85^\circ \text{ A}$$

Finally,

$$\begin{aligned} \mathbf{V}_0 &= 4(\mathbf{I}_1 - \mathbf{I}_2) \\ &= 4\left(\frac{20 + \mathrm{j}4}{6 + \mathrm{j}1} - 2\right) \\ &= 5.42\angle 4.57^\circ \text{ V} \end{aligned}$$

(b) In applying superposition to this problem, we consider each source acting alone. If we zero the current source, i.e., replace it with an open circuit, the circuit we obtain is shown in Fig. S7.4(b).

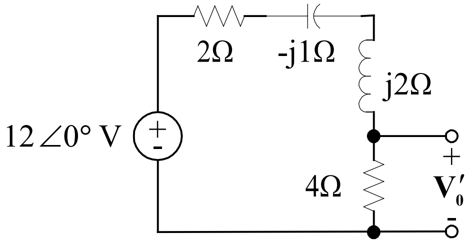

Fig. S7.4(b)

Using voltage division

$$\begin{aligned} \mathbf{V}_0' &= 12\left(\frac{4}{4 + \mathrm{j}2 + 2 - \mathrm{j}1}\right) \\ &= \frac{48}{6 + \mathrm{j}1} \text{V} \end{aligned}$$

Now, if we zero the voltage source, i.e., replace it with a short circuit, we obtain the circuit in Fig. S7.4(c).

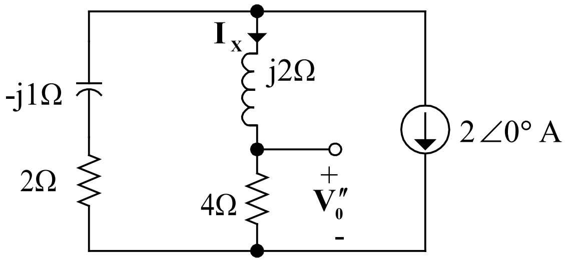

Fig. S7.4(c)

Employing current division, the current $\mathbf{I}_X$ is

$$\mathbf{I}_X = -2\angle 0°\left(\frac{2-j}{2-j+4+j2}\right) = \frac{-4+j2}{6+j1}\ \mathrm{A}$$

Then,

$$\mathbf{V}_0'' = 4\mathbf{I}_X = \frac{-16+j8}{6+j1}$$

And finally,

$$\begin{aligned}\mathbf{V}_0 &= \mathbf{V}_0' + \mathbf{V}_0'' \\ &= \frac{48}{6+j1} + \frac{-16+j8}{6+j1} \\ &= \frac{32+j8}{6+j1} \\ &= 5.42\angle 4.57°\ \mathrm{V}\end{aligned}$$

(c) In applying Thevenin's Theorem, we first break the network at the load and determine the open-circuit voltage as shown in Fig. S7.4(d).

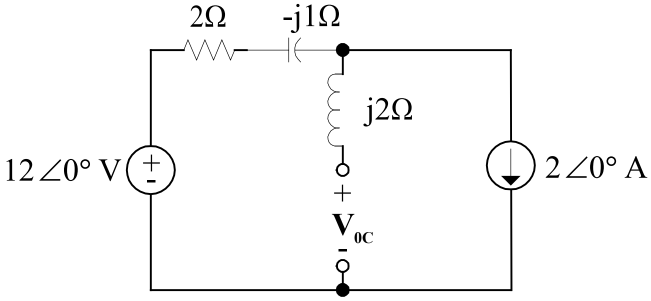

Fig. S7.4(d)

Note that there exists only one closed path and the current in it must be 2∠0° A. Note also that there is no current in the inductor and therefore no voltage across it. Hence $\mathbf{V}_{OC}$ is also the voltage across the current source. Hence,

$$\begin{aligned}\mathbf{V}_{0C} &= 12 - 2(2 - j)\\ &= 8 + j2 \text{ V}\end{aligned}$$

The Thevenin equivalent impedance found by zeroing the independent sources and looking into the network at the terminals of the load can be determined from the circuit in Fig. S7.4(e).

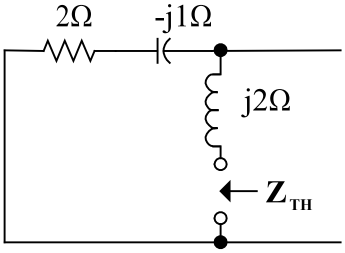

Fig. S7.4(e)

This network indicates that

$$\begin{aligned}\mathbf{Z}_{TH} &= 2 - j1 + j2\\ &= 2 + j1\Omega\end{aligned}$$

If we now form the Thevenin equivalent circuit and re-connect the load, we obtain the network in Fig. S7.4(f).

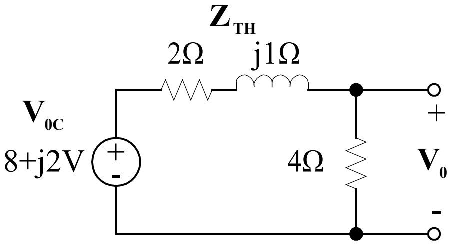

Fig. S7.4(f)

Applying voltage division yields

$$\begin{aligned}\mathbf{V}_0 &= (8 + j2)\left(\frac{4}{4 + 2 + j1}\right)\\ &= \frac{32 + j8}{6 + j1}\\ &= 5.42\angle 4.57^\circ \text{ V}\end{aligned}$$