

CHAPTER 11 PROBLEMS

- 11.1 Sketch the bode plot for the following network function

$$H(j\omega) = \frac{36(0.5j\omega + 1)}{(j\omega)^2(0.02j\omega + 1)}$$

- 11.2 Sketch the bode plot for the following network function

$$H(j\omega) = \frac{250j\omega(j\omega + 10)}{(j\omega + 1)(j\omega + 50)(j\omega + 100)}$$

- 11.3 Given the magnitude characteristic for the network function shown in Fig. 11.3, find the expression for $H(j\omega)$

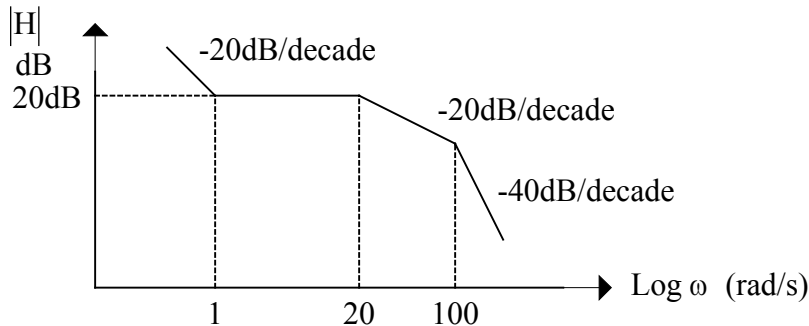


Fig. 11.3

- 11.4 Given the series circuit shown in Fig. 11.4, determine the following parameters: ω_0 , Q and the BW. If the resistance is changed to 0.1Ω , what is the impact on these parameters.

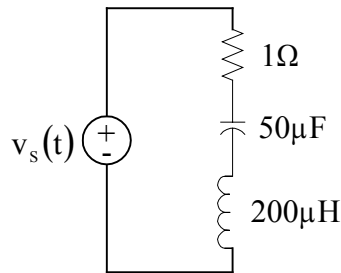


Fig. 11.4

Sketch the frequency characteristic for the two values of R . What conclusion can be drawn from these two characteristics.

- 11.5 The network in Fig. 11.5 operates as a band pass filter. (a) Determine the transfer function for the network, (b) find the upper and lower cut off frequencies and the band width and (c) sketch the magnitude characteristic for this transfer function.

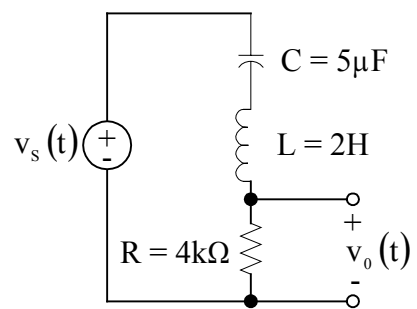


Fig. 11.5

CHAPTER 11 SOLUTIONS

- 11.1 First of all, we note that all the poles and zeros are in the standard form, e.g, the simple pole and zero are each in the form $(j\omega\tau + 1)$. At low frequencies the controlling term is the double pole at the origin. This term provides an initial slope for the magnitude characteristic of -40dB/decade . Furthermore, this initial slope will intersect the 0dB line at $\omega = \sqrt{36} = 6 \text{ rad/s}$. However, before this initial slope intersects the 0dB line, we encounter the break frequency of the zero at $\omega = \frac{1}{\tau} = \frac{1}{0.5} = 2 \text{ rad/s}$. This term adds a slope of $+20\text{dB/decade}$ to the magnitude characteristic and thus the composite characteristic changes from -40dB/decade to -20dB/decade . This characteristic maintains this slope until another break frequency is encountered. The remaining pole has a break frequency at $\omega = \frac{1}{\tau} = \frac{1}{0.02} = 50 \text{ rad/s}$. This term adds a slope of -20dB/decade to the magnitude characteristic, and since there are no more poles or zeros in the network function, the final slope of the magnitude characteristic is -40dB/decade . The composite magnitude characteristic is shown in Fig. S11.1(a).

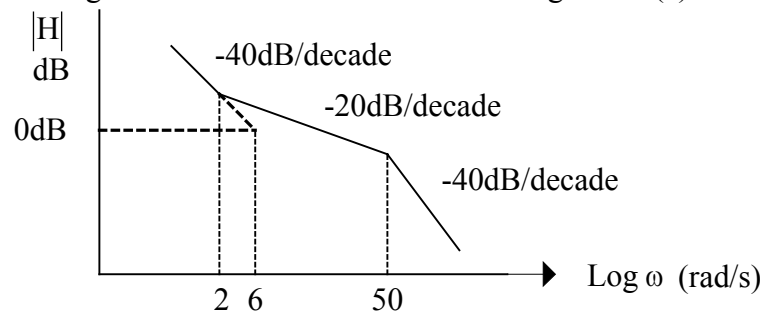


Fig. S11.1(a)

The composite phase characteristic for this network function is shown in Fig. S11.1(b).

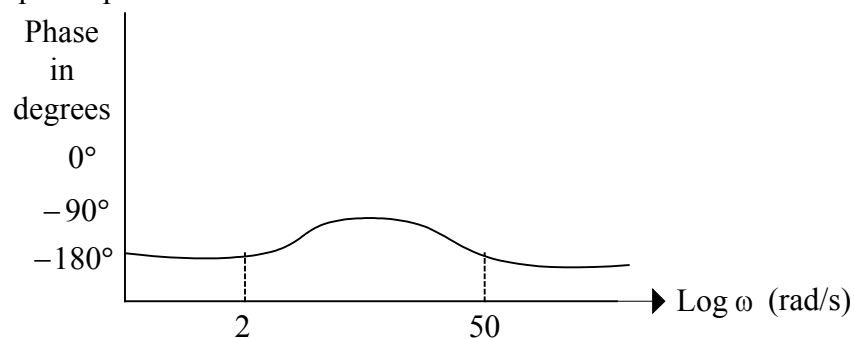


Fig. S11.1(b)

Once again, the initial phase, at low frequencies, is controlled by the double pole at the origin that has a constant phase of -180° . The phase for the zero is an arc tangent curve that provides 45° of phase at the break frequency, $\omega = 2 \text{ rad/s}$. As the frequency increases beyond the break frequency this term provides 90° of phase so the composite curve approaches -90° of phase. As the frequency increases further, we encounter the

simple pole which provides -45° of phase at its break frequency and finally -90° of phase at higher frequencies. Thus the composite phase starts at -180° , moves toward -90° because of the presence of the zero and finally ends up back at -180° because of the last pole.

- 11.2 We begin the analysis by putting all the terms of the network function in standard form. The function then becomes

$$H(j\omega) = \frac{0.5j\omega(0.1j\omega + 1)}{(j\omega + 1)(0.02j\omega + 1)(0.01j\omega + 1)}$$

At low frequencies the magnitude characteristic is controlled by the zero at the origin. This term provides an initial slope of $+20\text{dB/decade}$ and it will intersect the 0dB line at $\omega = \frac{1}{0.5} = 2 \text{ rad/s}$. Prior to reaching this frequency we encounter the break frequency of

the pole $(j\omega + 1)$ which occurs at $\omega = \frac{1}{\tau} = \frac{1}{1} = 1 \text{ rad/s}$. This term adds a slope of $-$

20dB/decade to the magnitude characteristic and therefore the composite characteristic has a net slope of $-20 + 20 = 0\text{dB/decade}$, i.e., the composite characteristic is flat until it encounters another break frequency. The next break frequency is due to the simple zero with break frequency at $\omega = \frac{1}{0.1} = 10 \text{ rad/s}$. At this point, the composite curve changes slope to $+20\text{dB/decade}$. The remaining two terms in the network function are poles with break frequencies at $\omega = \frac{1}{0.02} = 50 \text{ rad/s}$ and $\omega = \frac{1}{0.01} = 100 \text{ rad/s}$. Since each adds a

slope of -20dB/decade , the composite characteristic shifts from $+20\text{dB/decade}$ to 0dB/decade and then to -20dB/decade . The total composite characteristic is shown in Fig. S11.2(a).

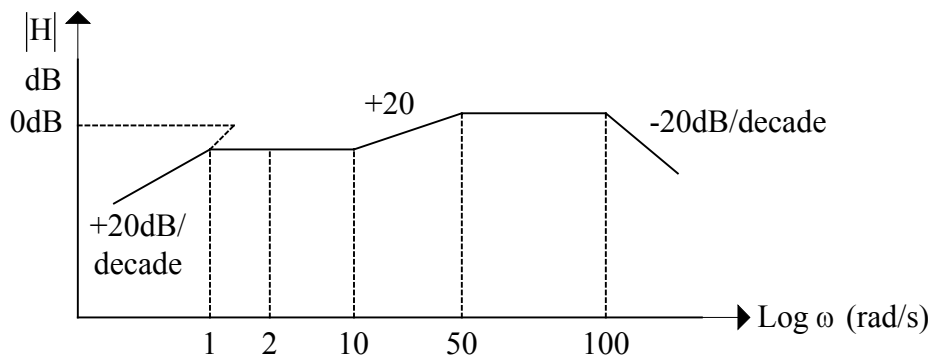


Fig. S11.2(a)

The composite phase characteristic for the network function is shown in Fig. S11.2(b).

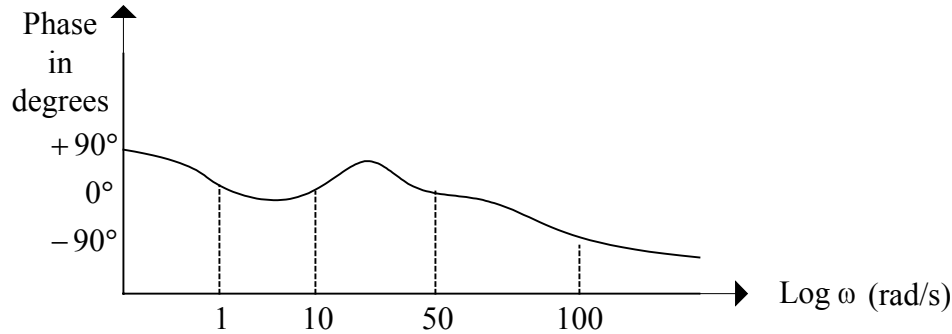


Fig. S11.2(b)

At low frequencies, the initial phase is $+90^\circ$ due to the zero at the origin. The first break frequency encountered is due to the pole term $(j\omega + 1)$ with break frequency at $\omega = 1$ rad/s. Thus the phase shifts toward 0° on an arc tangent curve that provides -45° of phase at $\omega = 1$ rad/s. The phase proceeds toward 0° until it encounters the zero with a break frequency of $\omega = \frac{1}{0.1} = 10$ rad/s. This term shifts the phase toward $+90^\circ$ going through $+45^\circ$ at the break frequency. The two remaining poles shift the composite phase back to 0° and finally to -90° as the characteristic indicates.

- 11.3 Examining the magnitude characteristic we note that at low frequencies the characteristics has an initial slope of -20dB/decade indicating a single pole at the origin. Furthermore, this initial slope passes through the 20dBs at $\omega = 1$ rad/s. Since the slope is -20dB/decade , this initial slope will cross the 0dB line at $\omega = 10$ rad/s. Therefore, the constant term, i.e., gain, in the network function is 10. Since the slope changes at $\omega = 1$ rad/s from -20dB/decade to 0dB/decade , there is a simple zero at this break frequency. At $\omega = 20$ rad/s, the slope changes again. This time the slope shifts from 0dB/decade to -20dB/decade indicating the presence of a simple pole with break frequency $\omega = 20$ rad/s. Finally, there is another simple pole with break frequency $\omega = 100$ rad/s. Therefore, the composite network function is

$$H(j\omega) = \frac{10(j\omega + 1)}{(j\omega)\left(\frac{j\omega}{20} + 1\right)\left(\frac{j\omega}{100} + 1\right)}$$

- 11.4 For this network, the resonant frequency is

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{(200 \times 10^{-6})(50 \times 10^{-6})}} \\ &= 10,000 \text{ rad/s}\end{aligned}$$

The quality factor is

$$\begin{aligned} Q &= \frac{\omega_0 L}{R} \\ &= \frac{(10^4)(200 \times 10^{-6})}{1} \\ &= 2 \end{aligned}$$

And the bandwidth is

$$\begin{aligned} \text{BW} &= \frac{\omega_0}{Q} \\ &= \frac{10^4}{2} \\ &= 5000 \text{ rad/s} \end{aligned}$$

If the resistance, R , is now changed from 1Ω to 0.1Ω the resonant frequency is unaffected. However, the Q changes to

$$\begin{aligned} Q &= \frac{\omega_0 L}{R} \\ &= \frac{(10^4)(200 \times 10^{-6})}{0.1} \\ &= 20 \end{aligned}$$

And the bandwidth is

$$\begin{aligned} \text{BW} &= \frac{\omega_0}{Q} \\ &= \frac{10^4}{20} \\ &= 500 \text{ rad/s} \end{aligned}$$

A sketch of the two frequency characteristics is shown in Fig. S11.4.

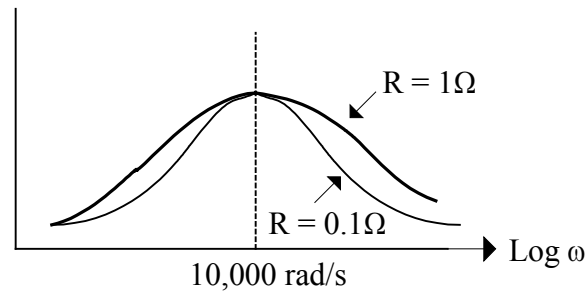


Fig. S11.4

Note that the higher value of Q , i.e., lower value of R , produces a more selective circuit with a much smaller bandwidth.

11.5 (a) Using voltage division, we can express the output as

$$\mathbf{V}_o = \left[\frac{R}{R + j\omega L + \frac{1}{j\omega C}} \right] \mathbf{V}_s$$

or

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

And therefore

$$\left| \frac{\mathbf{V}_o}{\mathbf{V}_s} \right| = \frac{RC\omega}{\left[(RC\omega)^2 + \left(\omega^2 LC - 1 \right)^2 \right]^{\frac{1}{2}}}$$

(b) The upper and lower cut off frequencies are the roots of the characteristic equation, i.e., the denominator of the transfer function.

At the lower cut off frequency

$$\omega^2 LC - 1 = -RC\omega$$

or

$$\omega^2 + \frac{R}{L}\omega - \omega_0^2 = 0$$

where, of course, $\omega_0^2 = \frac{1}{LC}$. With the component values, this function becomes

$$\omega^2 + 2000\omega - 10^5 = 0$$

Solving for ω_{LO} , we obtain

$$\begin{aligned}\omega_{LO} &= \frac{-2000 + \sqrt{(2000)^2 + 4 \times 10^5}}{2} \\ &= 48.8 \text{ rad/s}\end{aligned}$$

At the upper cut off frequency

$$\omega^2 LC - 1 = +RC\omega$$

or

$$\omega^2 - \frac{R}{L}\omega - \omega_0^2 = 0$$

and ω_{HI} is

$$\begin{aligned}\omega_{HI} &= \frac{2000 + \sqrt{(2000)^2 + 4 \times 10^5}}{2} \\ &= 2048.8 \text{ rad/s}\end{aligned}$$

Therefore, the bandwidth is

$$\begin{aligned}BW &= \omega_{HI} - \omega_{LO} = \frac{R}{L} \\ &= 2048.8 - 48.8 = \frac{4000}{2} \\ &= 2000 \text{ rad/s}\end{aligned}$$

(c) Since the resonant frequency is

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{2 \times 5 \times 10^{-6}}} \\ &= 316.23 \text{ rad/s}\end{aligned}$$

The magnitude characteristic for the function is shown in Fig. S11.5.

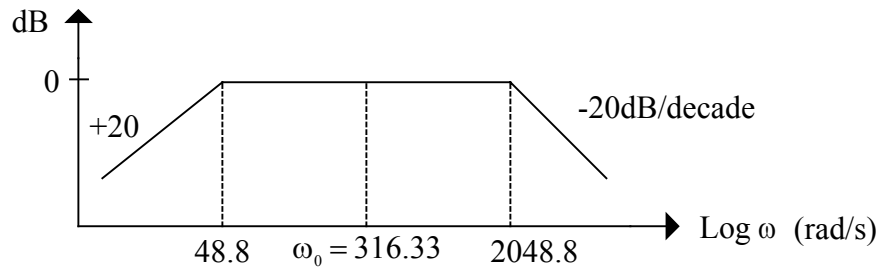


Fig. S11.5