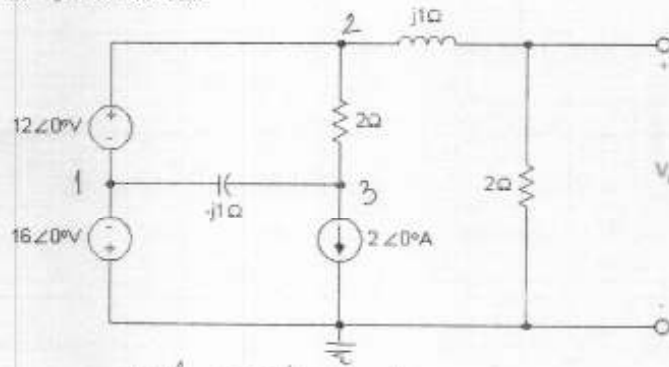


Find  $V_o$  in the network.



Let's use nodal analysis :

$$V_1 = -16\angle 0^\circ = -16 \text{ V}$$

$$V_2 = 12\angle 0^\circ - 16\angle 0^\circ = -4 \text{ V}$$

$$2\angle 0^\circ + \frac{V_3 - V_1}{-j1} + \frac{V_3 - V_2}{2} = 0$$

$$2 + j(V_3 + 16) + \frac{1}{2}(V_3 + 4) = 0$$

$$4 + j2V_3 + j32 + \frac{V_3}{2} + 4 = 0$$

$$V_3(1 + j2) + (8 + j32) = 0$$

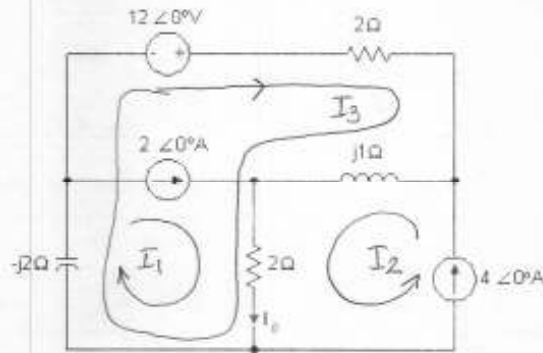
$$V_3 = -\frac{8 + j32}{1 + j2}$$

$$V_2 = -4 \quad V_o = \frac{2}{2 + j1} V_2$$

$$V_o = -\frac{8}{2 + j1} \frac{2 - j1}{2 - j1} = -8 \frac{2 - j1}{5}$$

$$V_o = -\frac{8}{5}(2 - j1)$$

Find  $I_0$  in the network using loop analysis.



$$I_1 = 2 \angle 0^\circ = 2$$

$$I_2 = 4 \angle 0^\circ = 4$$

$$-12 + 2(I_3) + j1(I_3 + I_2) + 2(I_1 + I_3 + I_2) - j^2(I_1 + I_2) = 0$$

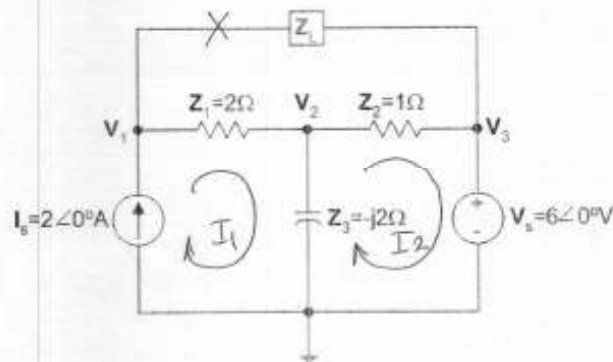
$$-12 + (2 + j1 + 2 - j2)I_3 + (j1 + 2)I_2 + (2 - j2)I_1 = 0$$

$$-12 + (4 - j1)I_3 + (j1 + 2)4 + (2 - j2)2 = 0$$

$$\cancel{-12} + \cancel{j4} + \cancel{8} + 4 - \cancel{j4} + (4 - j1)I_3 = 0 \quad I_3 = 0$$

$$I_0 = I_1 + I_2 + I_3 = 2 + 4 + 0 = 6 \text{ A}$$

Determine the impedance  $Z_L$  for maximum power transfer and the maximum power transferred. Use thevenin's theorem when solving this problem.



The best way to solve this problem is to use Thevenin's theorem.

$$I_1 = 2 \angle 0^\circ = 2$$

$$-j2(I_2 - I_1) + I_2 + 6 \angle 0^\circ = 0$$

$$-j2I_2 + j2 \times 2 + I_2 + 6 = 0$$

$$(1 - j2)I_2 + (6 + j4) = 0 \Rightarrow I_2 = -\frac{6 + j4}{1 - j2}$$

$$V_{Th} = 2I_1 + 1I_2 = 4 + \frac{6 + j4}{1 - j2} = \frac{4(1 - j2) - 6 - j4}{1 - j2}$$

$$V_{Th} = \frac{4 - j8 - 6 - j4}{1 - j2} = \frac{-2 - j12}{1 - j2} = -\frac{2(1 + j6)}{1 - j2}$$

$$Z_{Th} = 2 + \frac{1(-j2)}{1 - j2} = \frac{2 - j4 - j2}{1 - j2} = \frac{2 - j6}{1 - j2}$$

$$Z_L = Z_{Th}^* = \frac{2+j6}{1+j2}$$



$$I = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$V_L = \frac{Z_L}{Z_L + Z_{Th}} V_{Th}$$

$$P = VI^* = \frac{Z_L V_{Th}}{(Z_L + Z_{Th})} \cdot \frac{V_{Th}^*}{(Z_L + Z_{Th})^*} = \frac{Z_L |V_{Th}|^2}{|Z_L + Z_{Th}|^2}$$

$$Z_L + Z_{Th} = \frac{2+j6}{1+j2} + \frac{2-j6}{1-j2} = \frac{(2+j6)(1-j2) + (2-j6)(1+j2)}{1+4}$$

$$= \frac{2+j6-j4+12 + 2-j6+j4+12}{5} = \frac{28}{5}$$

$$|V_{Th}|^2 = \frac{4(1+36)}{1+4} = \frac{4 \times 37}{5}$$

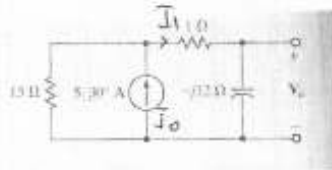
$$Z_L = \frac{2+j6}{1+j2} \cdot \frac{1-j2}{1-j2} = \frac{2+j6-j4+12}{1+4} = \frac{14+j2}{5}$$

$$P = \frac{14+j2}{5} \cdot \frac{148}{5} \left(\frac{37}{28}\right)^2 = \frac{148}{28^2} (14+j2)$$

Power absorbed is the real power only, namely.

$$P_R = \frac{1}{2} \frac{148 \times 14}{28 \times 28} = \frac{174}{2 \times 28} = \frac{37}{28} \text{ W}$$

Find the frequency-domain voltage  $V_0$ , as shown in the figure below.



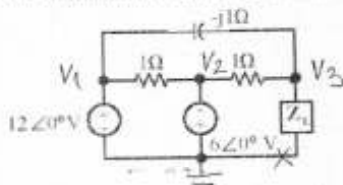
$$I_1 = \frac{15}{15 + 1 - j12} 5 \angle 30^\circ = \frac{15}{16 - j12} 5 \angle 30^\circ = \frac{15}{4} \frac{5 \angle 30^\circ}{4 - j3}$$

$$V_0 = -j12 I_1 = -j12 \frac{15}{4} \frac{5 \angle 30^\circ}{4 - j3} = -j \frac{225}{4 - j3} \angle 30^\circ$$

$$V_0 = \frac{225 \angle 30^\circ - 90^\circ}{\sqrt{16 + 9} \angle -\tan^{-1}(\frac{3}{4})} = \frac{225}{5} \angle -60^\circ + \tan^{-1}(\frac{3}{4})$$

$$V_0 = 45 \angle \tan^{-1}(\frac{3}{4}) - 60^\circ$$

Given the circuit in Fig. 9.2, determine the impedance  $Z_L$  for maximum average power transfer and the value of the maximum average power transferred to this load.



Again, we have to use Thevenin's theorem.

$$V_1 = 12\angle 0^\circ = 12V$$

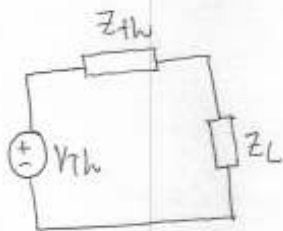
$$V_2 = -6\angle 0^\circ = -6V$$

$$\frac{V_3 - V_1}{-j1} + \frac{V_3 - V_2}{1} = 0 \quad \frac{V_3 - 12}{-j1} + (V_3 + 6) = 0$$

$$V_3 - 12 = j(V_3 + 6)$$

$$V_3(1 - j) = 12 + j6 \quad V_3 = \frac{12 + j6}{1 - j1} = V_{Th}$$

$$Z_{Th} = 1 \parallel (-j1) = \frac{1 \cdot (-j1)}{1 - j1} = -j \frac{1}{1 - j1}$$



$$Z_L = Z_{Th}^* = j \frac{1}{1 + j1}$$

$$I = \frac{V_{Th}}{Z_L + Z_{Th}} = \frac{\frac{12 + j6}{1 - j1}}{j \frac{1}{1 + j1} - j \frac{1}{1 - j1}} =$$

$$= \frac{(12 + j6)(1 + j)}{j(1 - j1) - j} = \frac{(12 + j6)(1 + j)}{j + 1 - j}$$

$$|I_0|^2 = \frac{|V|^2 (1 + 144\omega^2 C^2)}{(24 - 12\omega^2 LC)^2 + (\omega L + 288\omega C)^2}$$

$$I_1^2 = \frac{|V|^2}{(24 - 12\omega^2 LC)^2 + (\omega L + 288\omega C)^2}$$

1. when  $V = 60$

$$P_1 = \left( 24 \cdot \frac{60^2}{24^2} + \frac{1}{2} \cdot 12 \cdot \frac{60^2}{24^2} \right)$$

$$P_1 = \frac{60^2}{24} \left( 1 + \frac{1}{2} \right)$$

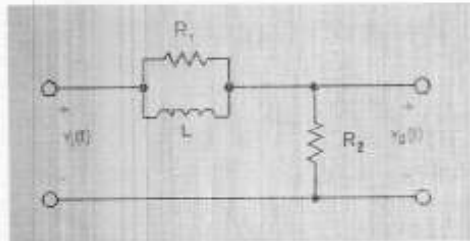
2. when  $V = 36 \angle 45^\circ$

$$P_2 = \frac{24 \cdot 36^2 (1 + 144\omega_0^2 C^2)}{(24 - 12\omega_0^2 LC)^2 + (\omega_0 L + 288\omega_0 C)^2} \cdot \frac{1}{2} +$$

$$+ \frac{1}{2} \cdot 12 \cdot \frac{36^2}{(24 + 12\omega_0^2 LC)^2 + (\omega_0 L + 288\omega_0 C)^2}$$

3. when  $V = 24 \angle 60^\circ$  and  $\omega = 2\omega_0$  we get similar to the above expression under 2 except that the voltage is 24 and the frequency is  $2\omega_0$ .

Determine the type of the filter network shown in the figure below, by determining the voltage transfer function.



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + \frac{sL \cdot R_1}{R_1 + sL}} = \frac{(R_1 + sL) R_2}{R_1 R_2 + sL(R_1 + R_2)}$$

$$H_o(s) = \frac{R_2(R_1 + sL)}{R_1 R_2 + sL(R_1 + R_2)}$$

$$|H_o(j\omega)|^2 = \frac{|R_2(R_1 + j\omega L)|^2}{(R_1 R_2)^2 + \omega^2 L^2 (R_1 + R_2)^2} = \frac{R_2^2 (R_1^2 + (\omega L)^2)}{(R_1 R_2)^2 + (\omega L)^2 (R_1 + R_2)^2}$$

$$1. \quad \omega \rightarrow 0 \quad |H_o(0)|^2 = \frac{R_1^2 R_2^2}{(R_1 R_2)^2} = 1$$

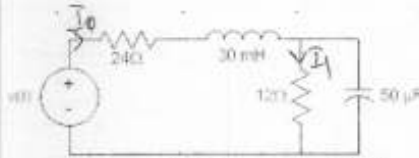
$$2. \quad \omega \rightarrow \infty \quad |H_o(\infty)|^2 = \frac{R_2^2 \cancel{\omega^2}}{\cancel{\omega^2} (R_1 + R_2)^2} = \frac{R_2^2}{(R_1 + R_2)^2} < 1$$

when  $R_2 \rightarrow 0$   $|H_o(\infty)|^2 = 0$  ~~low~~ pass filter

when  $R_2 \rightarrow \infty$   $|H_o(\infty)|^2 = 1$  all pass filter

Find the average power absorbed by the network if:

$$v(t) = 60 + 36\cos(377t + 45^\circ) + 24\cos(754t + 60^\circ)$$



$$V(t) = 60 + 36 \angle 45^\circ + 24 \angle 60^\circ$$

$$I_0 = \frac{V}{24 + j\omega L + \frac{12 \cdot \frac{1}{j\omega C}}{12 + \frac{1}{j\omega C}}} = \frac{V(1 + j12\omega C)}{(24 + j\omega L)(1 + j12\omega C) + 12}$$

$$I_0 = \frac{V(1 + j12\omega C)}{24 + j\omega L + \frac{j12 \times 24 \omega C - 12\omega^2 LC}{\frac{24}{288}}}$$

$$I_0 = \frac{V(1 + j12\omega C)}{24 - 12\omega^2 LC + j(\omega L + 288\omega C)}$$

$$I_1 = I_0 \frac{\frac{1}{j\omega C}}{12 + \frac{1}{j\omega C}} = \frac{I_0}{1 + j12\omega C}$$

$$I_1 = \frac{1}{1 + j12\omega C} \cdot \frac{V(1 + j12\omega C)}{(24 - 12\omega^2 LC) + j(\omega L + 288\omega C)}$$

$$I_1 = \frac{V}{(24 - 12\omega^2 LC) + j(\omega L + 288\omega C)}$$

$$I = 12 + j6 + j12 - 6 = 6 + j18$$

$$P_L = Z_L I I^* = Z_L |I|^2 = \frac{j}{1+j} (36 + 18^2)$$

$$P_L = \frac{j(1-j)}{1+j} (36 + 18^2) \frac{1}{2} = \frac{j+1}{2} (36 + 18^2)$$

$$P_{\text{Dissipated}} = \frac{1}{2} \frac{1}{2} (36 + 18^2) = \frac{1}{2} (18 + 3 \times 18) = 18 \frac{(10)}{2} = \frac{180 \text{ W}}{2}$$

$$P_{\text{Dissipated}} = 90 \text{ W}$$