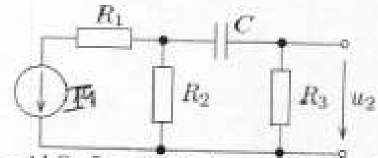
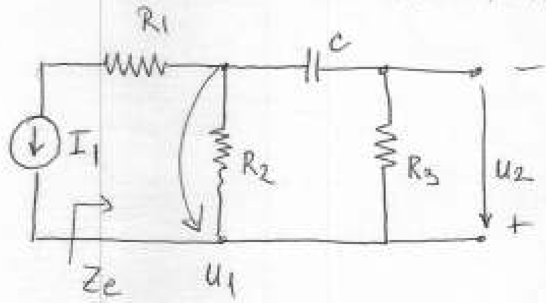


$R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $R_3 = 4\,\mathrm{k\Omega}$, $C = 1\,\mu\mathrm{F}$,
$U_0 = 5\,\mathrm{V}$, $U_{m1} = 10\,\mathrm{V}$, $U_{m3} = 2\,\mathrm{V}$,
$\omega_0 = 1000\,\mathrm{s}^{-1}$, $\varphi_1 = 1/4\,\pi$, $\varphi_3 = -1/3\,\pi$



$$Z_e = \frac{R_2\left(R_3 + \frac{1}{j\omega C}\right)}{R_2 + R_3 + \frac{1}{j\omega C}} = \frac{R_2(1 + j\omega C R_3)}{1 + j\omega C(R_2 + R_3)}$$

$$u_1 = Z_e I_1 \qquad U_2 = \frac{R_3}{R_3 + \frac{1}{j\omega C}} u_1 = \frac{j\omega C R_3}{1 + j\omega C R_3} u_1$$

$$u_2 = \frac{j\omega C R_3}{1 + j\omega C R_3} \cdot \frac{R_2(1 + j\omega C R_3)}{1 + j\omega C(R_2 + R_3)} I_1$$

Where $I_1 = 5 + 10\sin(\omega_0 t + \pi/4) + 2\sin(3\omega_0 t - \pi/3)$

DC component $\omega = 0$ $\quad$ $U_{2DC} = 0$ capacitor prevents current flow

1st AC component
$I_1 = 10\angle\pi/4 \ (\omega_0)$

2nd AC component
$I_2 = 2\angle{-\pi/3} \ (3\omega_0)$

$$U_2 = \frac{j\omega 10^{-6} 4K}{1 + j\omega 10^{-6} 4K} \cdot \frac{2K(1 + j\omega 10^{-6} \cdot 4K)}{1 + j\omega 10^{-6}(2K + 1K)} I_1$$

$$= \frac{j4\omega \cancel{m}}{1 + j4\omega m} \frac{2\cancel{K}(1 + j\omega 4m)}{1 + j\omega \cdot 3m} I_1$$

$$= \frac{j4\omega}{1 + j4\omega m} \cdot \frac{2(1 + j4\omega m)}{1 + j3\omega m} I_1$$

when $\omega = \omega_o = 1K$ and $I_1 = 10 / \pi/4$

$$U_{2_\omega} = \frac{j4}{1 + j4} \frac{2(1 + j4)}{1 + j3} 10 \frac{\sqrt{2}}{2}(1 + j)$$

when $\omega = 3\omega_o = 3K$ and $I_2 = 2e^{-j\pi/3}$

$$U_{2(3\omega)} = \frac{j12}{1 + j12} \frac{2(1 + j12)}{1 + j9} 2e^{-j\pi/3}$$

Then

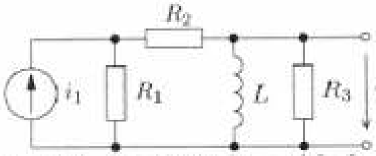
$$U_2 = U_{2\omega} + U_{2_{3\omega}}$$

$\downarrow$ convert each term in real space

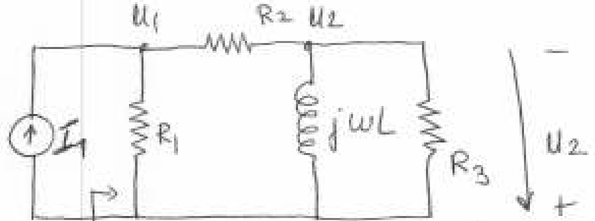
$$U_2(t) = U_{21}\cos(\omega_o t + \theta_1) + U_{22}\cos(3\omega_o t + \theta_3)$$

$$U_{2rms} = \sqrt{U_{21}^2 + U_{22}^2}$$

Circuits shown in the two figures below are supplied by the periodic non-sinusoidal current $i_1(t) = I_0 + I_{m1}\sin(\omega_0 t + \varphi_1) + I_{m3}\sin(3\omega_0 t + \varphi_3)$ and they are in the steady state. Find the output voltage $u_2(t)$ and its effective (rms) value U_2



$R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $R_3 = 4\,\mathrm{k\Omega}$, $L = 1\,\mathrm{H}$,
$I_0 = 5\,\mathrm{mA}$, $I_{m1} = 10\,\mathrm{mA}$, $I_{m3} = 2\,\mathrm{mA}$,
$\omega_0 = 1000\,\mathrm{s}^{-1}$, $\varphi_1 = 1/4\,\pi$, $\varphi_3 = -1/3\,\pi$



$$Z_e = \frac{R_1\left(R_2 + \frac{R_3 j\omega L}{R_3 + j\omega L}\right)}{R_1 + R_2 + \frac{R_3 j\omega L}{R_3 + j\omega L}} = \frac{R_1\left(R_2(R_3 + j\omega L) + R_3 j\omega L\right)}{(R_1 + R_2)(R_3 + j\omega L) + R_3 j\omega L}$$

$$Z_e = \frac{R_1 R_2 R_3 + j\omega L(R_3 R_1 + R_2 R_1)}{(R_1 + R_2)R_3 + j\omega L(R_1 + R_2 + R_3)}$$

$$U_1 = Z_e I_1$$

$$U_2 = -\frac{\frac{R_3 j\omega L}{R_3 + j\omega L}}{R_2 + \frac{R_3 j\omega L}{R_3 + j\omega L}} U_1 = -\frac{j\omega L R_3 U_1}{R_2(R_3 + j\omega L) + R_3 j\omega L} = \frac{-j\omega L R_3 U_1}{R_2 R_3 + j\omega L(R_2 + R_3)}$$

$$U_2 = -\frac{j\omega L R_3}{R_2 R_3 + j\omega L(R_2 + R_3)} \cdot \frac{R_1 R_2 R_3 + j\omega L(R_3 + R_2)R_1}{(R_1 + R_2)R_3 + j\omega L(R_1 + R_2 + R_3)} I_1$$

$$u_2 = -\frac{j\omega 4\cancel{K}}{8K^{\cancel{2}} + j\omega(4+2)\cancel{K}} \cdot \frac{8K^{\cancel{2}} + j\omega(6K)\cdot 1K}{3\cancel{K}\cdot 4K + j\omega(7\cancel{K})\,m} I_1$$

$$= \frac{-j4\omega}{8K + j6\omega} \cdot \frac{8K + j6\omega}{12 + j7\omega m} I_1$$

Now:

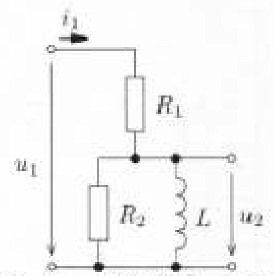
1) $\omega = \omega_0 = 1000\,s^{-1} = 1K \qquad I_1 = I_{m1} e^{j\varphi_1}$

$$\underset{\omega_0}{u_2} = -\frac{j4K}{8K + j6K} \frac{8K + j6K}{1 + j7} I_{m1} e^{j\varphi_1}$$

2) $\omega = 3\omega_0 = 3K\ s^{-1} \qquad I_1 = I_{m3} e^{j\varphi_3}$

$$\underset{3\omega_0}{U_2} = -\frac{j12K}{\cancel{8K + j18K}} \frac{\cancel{8K + j18K}}{12 + j21} I_{m3} e^{j\varphi_3} = -\frac{j12K}{12 + j21} I_{m3} e^{j\varphi_3}$$

convert each into real time and add real time quantities



$R_1 = 100\,\Omega$, $R_2 = 1000\,\Omega$, $L = 1\,\mathrm{H}$, $\cancel{U_0 = 5\,\mathrm{V}}$,
$\cancel{U_{m1} = 10\,\mathrm{V}}$, $\cancel{U_{m3} = 20\,\mathrm{V}}$, $\cancel{\omega_0 = 1000\,\mathrm{s}^{-1}}$,

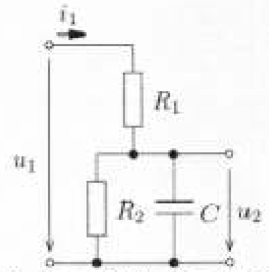
Transfer function calculation:

$$Z_e = \frac{R_2 j\omega L}{R_2 + j\omega L}$$

$$H(\omega) = \frac{U_2(\omega)}{U_1(\omega)} = \frac{Z_e}{R_1 + Z_e} = \frac{\frac{R_2 j\omega L}{R_2 + j\omega L}}{R_1 + \frac{R_2 j\omega L}{R_2 + j\omega L}} = \frac{j\omega L R_2}{R_1(R_2 + j\omega L) + j\omega L R_2}$$

$$H(\omega) = \frac{j\omega 1K}{0.1\cancel{K}(1K + j\omega) + j\omega \cdot 1\cancel{K}} = \frac{j\omega}{0.1K + j\omega(1 + 0.1)}$$

$$H(\omega) = \frac{j\omega}{0.1K + j\omega 1.1}$$



$R_1 = 100\,\Omega$, $R_2 = 500\,\Omega$, $C = 1\,\mu\mathrm{F}$, ~~$U_0 = 5\,\mathrm{V}$~~,
~~$U_{m1} = 10\,\mathrm{V}$, $U_{m2} = 20\,\mathrm{V}$, $\omega_0 = 1000\,s^{-1}$~~,
~~$\varphi_1 = 1/4\pi$, $\varphi_2 = -1/3\pi$~~

$$Ze = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega C R_2}$$

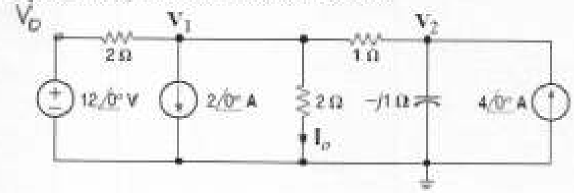
$$H(\omega) = \frac{U_2(\omega)}{U_1(\omega)} = \frac{Ze}{R_1 + Ze} = \frac{\frac{R_2}{1 + j\omega C R_2}}{R_1 + \frac{R_2}{1 + j\omega C R_2}}$$

$$H(\omega) = \frac{R_2}{R_1(1 + j\omega C R_2) + R_2} = \frac{R_2}{R_1 + R_2 + j\omega C R_1 R_2}$$

$$H(\omega) = \frac{0{,}5K}{0{,}6K + j\omega \cancel{10^{-6}}\, 0{,}1\cancel{K} \cdot 0.5\cancel{K}}$$

$$H(\omega) = \frac{0{,}5K}{0.6K + j0.05\omega}$$

Using nodal analysis, find $\mathbf{I}_0$ in the circuit shown bellow.



$V_0 = 12\angle 0$

Node 1: $\dfrac{V_1 - V_0}{2} + 2\angle 0 + \dfrac{V_1}{2} + \dfrac{V_1 - V_2}{1} = 0$

Node 2: $\dfrac{V_2 - V_1}{1} + \dfrac{V_2}{-j1} - 4\angle 0 = 0$

From Node 2 equation we have:

$V_2 - V_1 + jV_2 - 4 = 0 \Rightarrow V_2(1+j) - V_1 = 4$

$V_1 = V_2(1+j) - 4$

From Node 1 equation:

$V_1 - V_0 + 4 + V_1 + 2V_1 - 2V_2 = 0$

$V_1 - 12 + 4 + V_1 + 2V_1 - 2V_2 = 0$

$4V_1 - 2V_2 = 8 \Rightarrow 2V_1 - V_2 = 4 \Rightarrow$

$2(V_2(1+j) - 4) - V_2 = 0$

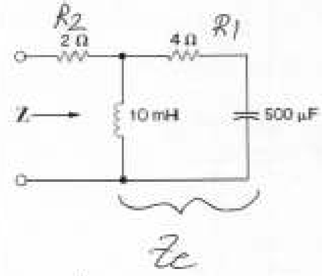
$2V_2(1+j) - 8 - V_2 = 0$

$V_2(1+2j) = 8 \qquad V_2 = \dfrac{8}{1+2j}$

$V_1 = \dfrac{8}{1+2j}(1+j) - 4$

$I_o = V_1/2$

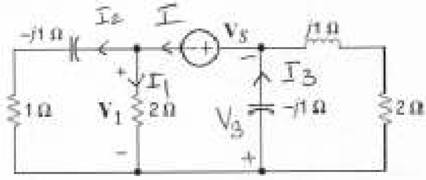
Find Z(jw) for f=60 Hz.



$$Z_e = \frac{j\omega L\left(R_1 + \frac{1}{j\omega C}\right)}{j\omega L + R_1 + \frac{1}{j\omega C}} = \frac{j\omega L\,(1 + j\omega C R_1)}{1 + j\omega C R_1 - \omega^2 LC}$$

$$Z = R_2 + Z_e = R_2 + \frac{j\omega L\,(1 + j\omega C R_1)}{1 - \omega^2 LC + j\omega C R_1}$$

Find V_S in the network, if $V_1 = 4$ V.



$V_1 = 4$ V $\quad I_1 = \frac{V_1}{2} = \frac{4}{2} = 2$

$$I_2 = \frac{V_1}{1-j1} = \frac{4}{1-j1} = \frac{4(1+j1)}{(1-j1)(1+j1)} = \frac{4(1+j1)}{2} = 2(1+j1)$$

$$I = I_1 + I_2 = 2 + 2(1+j1) = 4 + j2$$

$$I_3 = I\,\frac{2+j1}{2+\cancel{j1}-\cancel{j1}} = \frac{2+j1}{2}\,I$$

$$I_3 = \frac{2+j1}{\cancel{2}}\,\cancel{2}(2+j1) = (2+j1)^2$$

$$V_3 = -j1\,I_3 = -j(2+j1)^2 = $$
$$= -j(4+4j-1) = -j(3+4j) = 4-3j$$

KVL: $\quad -V_1 - V_S - V_3 = 0$

$$V_S = -V_1 - V_3 = -4 - (4-j3) = -8 + j3$$