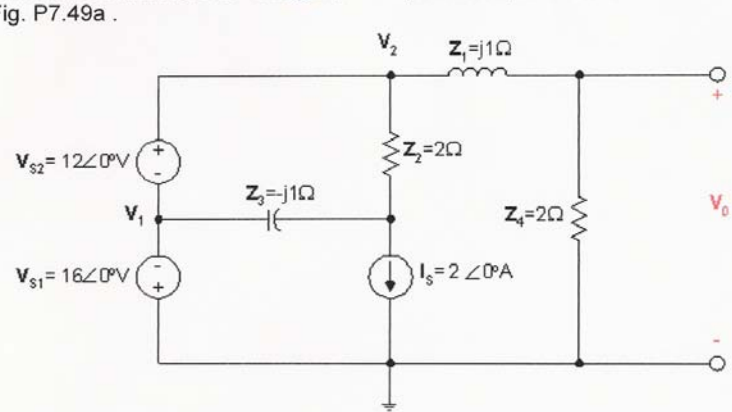
Solutions to #1

Problem 7.49

Problem **Hints** **Solution Guidelines** **Detailed Solution** **Final Answer**

1 Let us label the sources, node voltages, and impedances, and select a reference node as in Fig. P7.49a .



(Figure P7.49a)

2 First we note that the voltage source labeled $\mathbf{V}_{s1}$ is connected between node $\mathbf{V}_1$ and the reference node, thus:

$$\mathbf{V}_1 + \mathbf{V}_{s1} = 0$$
$$\mathbf{V}_1 = -16\angle 0^\circ \mathbf{V}$$

3 In addition, the voltage source labeled $\mathbf{V}_{s2}$ is connected between nodes $\mathbf{V}_1$ and $\mathbf{V}_2$. Therefore:

$$\mathbf{V_2} = \mathbf{V}_1 + \mathbf{V}_{s2} =$$
$$= -16\angle 0^\circ + 12\angle 0^\circ = -4\angle 0^\circ \text{ V}$$

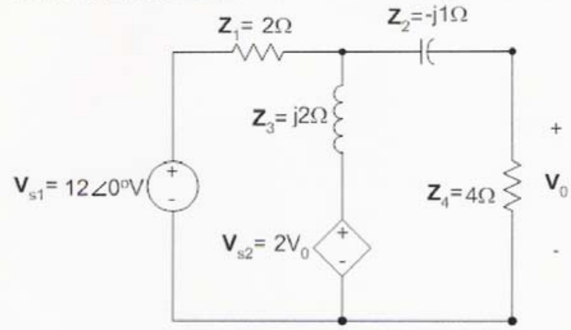
4 Knowing $\mathbf{V}_2$ we can use voltage division to find $\mathbf{V}_0$.

$$\mathbf{V}_0 = \mathbf{V}_2 \cdot \frac{\mathbf{Z}_4}{\mathbf{Z}_1 + \mathbf{Z}_4} = -4\left(\frac{2}{2+j1}\right)$$
$$\mathbf{V}_0 = 3.58\angle 153.43^\circ \text{ V}$$

Problem 7FE-3

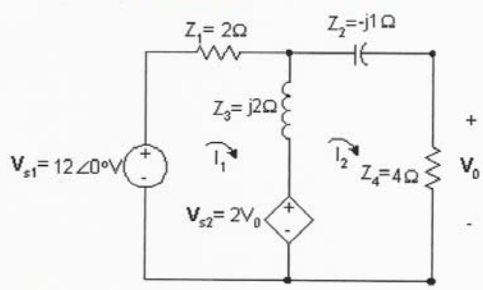
Problem | Hints | Solution Guidelines | Detailed Solution | Final Answer

1 Let us label the sources and the impedances as shown in Fig 7PEF-3a



(Figure 7PEF-3a)

2 Since the network has two meshes, we need two equations to determine all the currents. We will first assign one loop-current to each mesh as shown in Fig 7PFE-3b



3 The KVL equations for the meshes are:

$$-\mathbf{V}_{s1} + \mathbf{I}_1\,\mathbf{Z}_1 + (\mathbf{I}_1 - \mathbf{I}_2)\,\mathbf{Z}_3 + \mathbf{V}_{s2} = 0$$

$$\mathbf{I}_2\,\mathbf{Z}_2 + \mathbf{I}_2\,\mathbf{Z}_4 - \mathbf{V}_{s2} + (\mathbf{I}_2 - \mathbf{I}_1)\,\mathbf{Z}_3 = 0$$

4 Substituting the components values into the equations yields:

$$\mathbf{I}_1(2 + j2) + \mathbf{I}_2(-j2) = 12 - 2\,\mathbf{V}_0$$

$$\mathbf{I}_1(-j2) + \mathbf{I}_2(4 + j) = 2\,\mathbf{V}_0$$

5 $\mathbf{V}_0 = \mathbf{I}_2 \cdot \mathbf{Z}_4$ by Ohm's law, thus:

$V_0 = I_2 \cdot Z_4$ by Ohm's law, thus.

$$\mathbf{I}_1(2+j2) + \mathbf{I}_2(-j2) = 12 - 8\,\mathbf{I}_2$$
$$\mathbf{I}_1(-j2) + \mathbf{I}_2(4+j) = 8\,\mathbf{I}_2$$

6 And solving for $\mathbf{I}_2$ yields:

$$\mathbf{I}_2 = \frac{12}{5+j3} = 2.06\angle -30.96^\circ\,\mathrm{A}$$

7 Knowing the loop current we can find $\mathbf{V}_0$ using Ohm's law:

$$\mathbf{V}_0 = \mathbf{I}_2 \cdot \mathbf{Z}_4 = 2.06\angle -30.96^\circ \cdot 4 = 8.24\angle -30.96^\circ\,\mathrm{V}$$

5 The KVL equations for the above network are:

$$-\mathbf{V}_1 + R_1\,\mathbf{I}_1 + \mathbf{V}_S = 0$$
$$-\mathbf{V}_2 + R_2\,\mathbf{I}_2 = 0$$

6 Comparing the network to our standard circuit for mutually coupled inductors, we find that we must reverse the sign on $\mathbf{I}_1$ and $\mathbf{V}_2$.

Therefore, the equations that relate $\mathbf{V}_1$ and $\mathbf{V}_2$ to $\mathbf{I}_1$ and $\mathbf{I}_2$, in this case, are:

$$\mathbf{V}_1 = -j\omega L_1\,\mathbf{I}_1 + j\omega M\mathbf{I}_2$$
$$-\mathbf{V}_2 = j\omega L_2\,\mathbf{I}_2 - j\omega M\mathbf{I}_1$$

7 Combining the equations in (5) and (6) yields:

$$\left(j\omega L_1 + R_1\right)\mathbf{I}_1 + \left(-j\omega M\right)\mathbf{I}_2 + \mathbf{V}_S = 0$$
$$\left(-j\omega M\right)\mathbf{I}_1 + \left(R_2 + j\omega L_2\right)\mathbf{I}_2 = 0$$

8 Substituting the components values into (7) we obtain :

$$\left(200 + j400\right)\mathbf{I}_1 - j320\,\mathbf{I}_2 = -1$$
$$-j320\,\mathbf{I}_1 + \left(320 + j400\right)\mathbf{I}_2 = 0$$

9 Solving the equations yields:

10
$$\mathbf{I}_1 = 2.46\angle 143.1^\circ\text{mA}$$
$$\mathbf{I}_2 = 1.54\angle -178.2^\circ\text{mA}$$

11 Converting the currents back to time domain we obtain:

$$i_1(t) = 2.46\cos(100t + 143.1^\circ)\text{mA}$$
$$i_2(t) = 1.54\cos(100t - 178.24^\circ)\text{mA}$$

12 At $t = 2\text{ms}$, $100t = 0.2\text{rad}$ or 11.46°, and therefore,

$$i_1(t = 2\text{ms}) = 2.46\cos(11.46^\circ + 143.1^\circ) = -2.22\text{mA}$$
$$i_2(t = 2\text{ms}) = 1.54\cos(11.46^\circ - 178.24^\circ) = -1.5\text{mA}$$

13 Hence, the energy stored in the coupled inductors at $t = 2\text{ms}$ is

$$w(t)|_{t=2\text{ms}} = \frac{1}{2}L_1\left[i_1\left(t = 2\text{ms}\right)\right]^2 + \frac{1}{2}L_2\left[i_2\left(t = 2\text{ms}\right)\right]^2 - Mi_1\left(t = 2\text{ms}\right)i_2\left(t = 2\text{ms}\right)$$
$$= \frac{1}{2}\cdot 4\cdot\left[\left(-2.22\right)\times 10^{-3}\right]^2 + \frac{1}{2}\cdot 4\cdot\left[\left(-1.5\right)\times 10^{-3}\right]^2$$

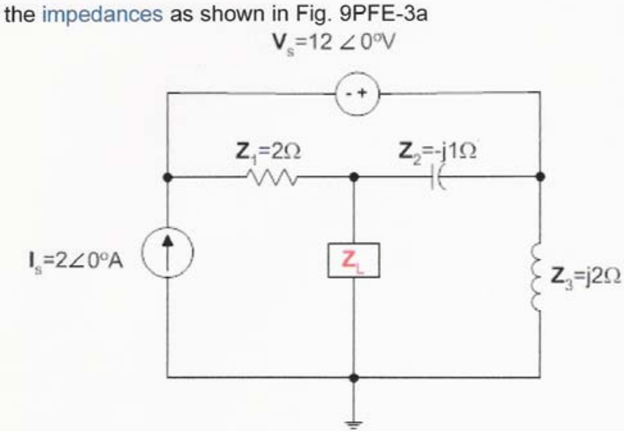
$$-3.2\cdot\left(-2.22\right)\times 10^{-3}\cdot\left(-1.5\right)\times 10^{-3}$$
$$= 3.7\times 10^{-6}\,\text{J} = 3.7\mu\text{J}$$

Problem 9FE-3

problem #4

Problem **Hints** **Solution Guidelines** **Detailed Solution** **Final Answer**

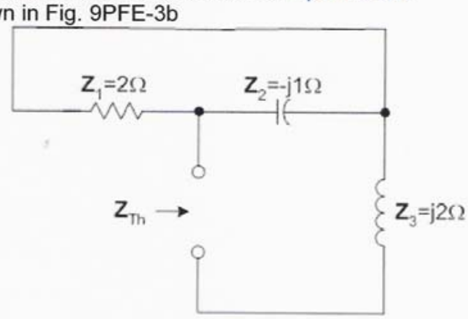
1 Let us label the impedances as shown in Fig. 9PFE-3a



(Figure 9PFE-3a)

2 To find the value of the load, $\mathbf{Z}_L$, for maximum average power transfer, we simply need to find the complex conjugate of the Thévenin equivalent impedance.

3 The Thévenin equivalent impedance is obtained by looking into the open-circuit terminals with all sources made zero. In this case, we replace the voltage source with a short circuit and the current source with an open-circuit.
The network is shown in Fig. 9PFE-3b



(Figure 9PFE-3b)

4 Combining the impedances yields:

$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1 \cdot \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$$= j2 + \frac{2 \cdot (-j1)}{2 + (-j1)} = 0.4 + j1.2\Omega$$

5 Hence, for maximum average power transfer, the value of the load is the complex conjugate of the Thévenin equivalent impedance hence

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 0.4 - j1.2\Omega$$