AW #3

CHAPTER 8 PROBLEMS

8.1 Find $\mathbf{V}_0$ in the network in Fig. 8.1.

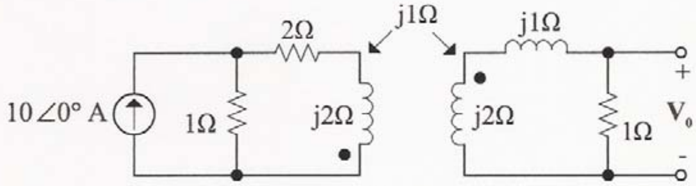

Fig. 8.1

8.2 Determine the impedance seen by the source in the circuit in Fig. 8.2.

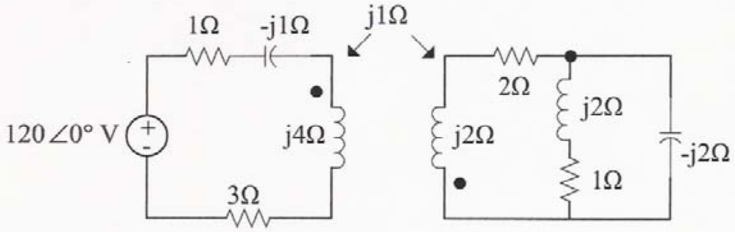

Fig. 8.2

8.3 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit in Fig. 8.3.

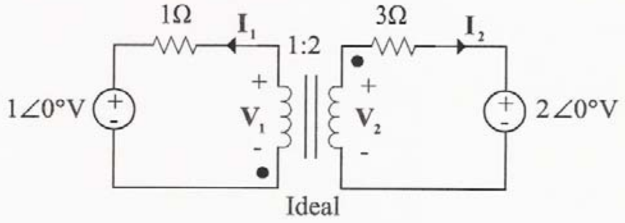

Fig. 8.3

8.4 Given the circuit in Fig. 8.3, determine the two networks obtained by replacing (a) the primary and the ideal transformer with an equivalent circuit and (b) the ideal transformer and the secondary with an equivalent circuit.

CHAPTER 8 SOLUTIONS

8.1 Our first step in the solution of this problem is to apply source transformation to the left-end of the network and transform the 10∠0°A source in parallel with the 1Ω resistor into a 10∠0°V source in series with the 1Ω resistor as shown in Fig. S8.1(a).

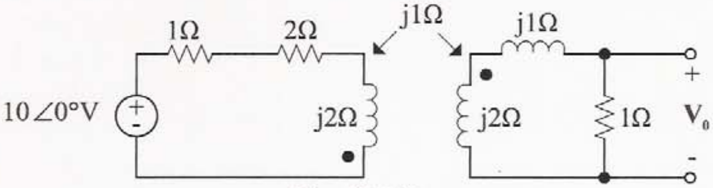

Fig. S8.1(a)

Let us redraw the network as shown in Fig. S8.1(b).

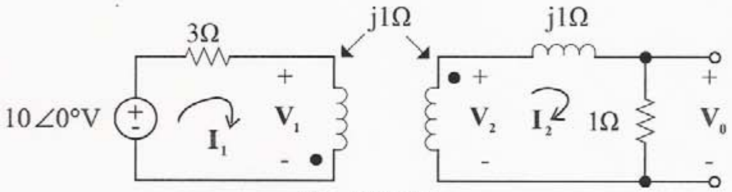

Fig. S8.1(b)

The equations for this network are

$$-10 + 3\mathbf{I}_1 + \mathbf{V}_1 = 0$$
$$-\mathbf{V}_2 + \mathbf{I}_2(1 + j1) = 0$$

We now write the equations for the mutually coupled coils. In order to force the variables in this circuit into our standard form for mutually coupled inductors, we must reverse the signs on $\mathbf{V}_1$, $\mathbf{I}_1$ and $\mathbf{I}_2$. Therefore, the equations that relate $\mathbf{V}_1$ and $\mathbf{V}_2$ to $\mathbf{I}_1$ and $\mathbf{I}_2$, in this case, are

$$-\mathbf{V}_1 = j2(-\mathbf{I}_1) + j1(-\mathbf{I}_2)$$
$$\mathbf{V}_2 = j2(-\mathbf{I}_2) + j1(-\mathbf{I}_1)$$

Combining the equations yields

$$(3 + j2)\,\mathbf{I}_1 + j1\,\mathbf{I}_2 = 10$$
$$j1\,\mathbf{I}_1 + (1 + j3)\,\mathbf{I}_2 = 0$$

Solving for $\mathbf{I}_1$ in the second equation and substituting it into the first equation yields

$$[(3 + j2)(-3 + j1) + j1]\,\mathbf{I}_2 = 10$$

or

$$\mathbf{I}_2 = \frac{10}{-11 - j2}$$
$$= -0.894\angle 10.3° \text{ A}$$

And finally

$$\mathbf{V}_0 = 1\mathbf{I}_2$$
$$= -0.894\angle 10.3° \text{ V}$$

8.2 Let us first determine the total impedance on the right side of the circuit as shown in Fig. S8.2(a).

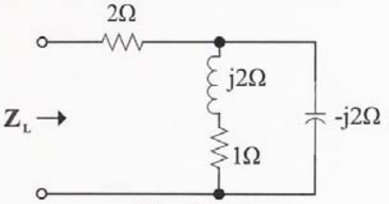

Fig. S8.2(a)

As the figure indicates

$$\mathbf{Z}_L = 2 + (1 + j2) \| (-j2)$$
$$= 2 + \frac{(1 + j2)(-j2)}{1 + j2 - j2}$$
$$= 6 - j2\Omega$$

The original network can now be redrawn in the following form shown in Fig. S8.2(b).

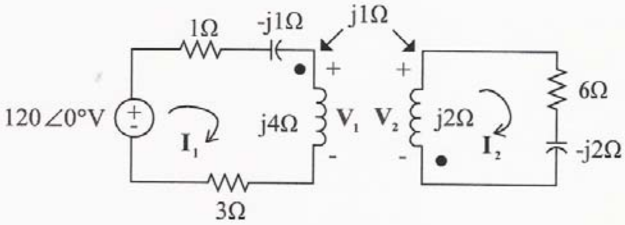

Fig. S8.2(b)

The two KVL equations for the network in Fig. S8.2(b) are

$$120 = (4 - j1)\,\mathbf{I}_1 + \mathbf{V}_1$$
$$\mathbf{V}_2 = (6 - j2)\,\mathbf{I}_2$$

In order to force the variables in this circuit into our standard form for mutually coupled inductors, we must reverse the sign on $\mathbf{V}_2$. Therefore, the equations that relate $\mathbf{V}_1$ and $\mathbf{V}_2$ to $\mathbf{I}_1$ and $\mathbf{I}_2$, in this particular case, are

$$\begin{aligned} \mathbf{V}_1 &= j4\,\mathbf{I}_1 + j1\,\mathbf{I}_2 \\ -\mathbf{V}_2 &= j2\,\mathbf{I}_2 + j1\,\mathbf{I}_1 \end{aligned}$$

Combining all of these equations results in the following two equations.

$$\begin{aligned} (4 + j3)\,\mathbf{I}_1 + j1\,\mathbf{I}_2 &= 120 \\ j1\,\mathbf{I}_1 + 6\,\mathbf{I}_2 &= 0 \end{aligned}$$

Solving the second equation for I_2 and substituting this value into the first equation yields

$$\left(4 + j3 + \frac{1}{6}\right)\mathbf{I}_1 = 120$$

Then, the impedance seen by the source is

$$\mathbf{Z}_s = \frac{120}{\mathbf{I}_1} = 4.167 + j3\Omega$$

8.3 The KVL equations for this network are

$$\begin{aligned} 1\angle 0° &= -\mathbf{I}_1(1) + \mathbf{V}_1 \\ \mathbf{V}_2 &= 3\mathbf{I}_2 + 2\angle 0° \end{aligned}$$

If we now force the variables in this circuit into our standard form for the ideal transformer, we must reverse the signs on $\mathbf{V}_1$ and $\mathbf{I}_2$. Therefore, the equations that relate $\mathbf{V}_1$ to $\mathbf{V}_2$ and $\mathbf{I}_1$ to $\mathbf{I}_2$, in this particular case, are

$$\begin{aligned} \frac{-\mathbf{V}_1}{\mathbf{V}_2} &= \frac{1}{2} \\ 1\mathbf{I}_1 + 2(-\mathbf{I}_2) &= 0 \end{aligned}$$

Solving the later equations for $\mathbf{V}_2$ and $\mathbf{I}_2$ and substituting these values into the first equations yields

$$\begin{aligned} 1 &= -\mathbf{I}_1 + \mathbf{V}_1 \\ -2\mathbf{V}_1 &= \frac{3}{2}\mathbf{I}_1 + 2 \end{aligned}$$

Solving these equations produces

$$\mathbf{I}_1 = 1.142\angle 180° \text{ A}$$
$$\mathbf{V}_1 = 0.142\angle 180° \text{ V}$$

Then, the transformer relationships yield

$$\mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1$$
$$\mathbf{V}_2 = -2\mathbf{V}_1$$

Therefore,

$$\mathbf{I}_2 = 0.571\angle 180° \text{ A}$$
$$\mathbf{V}_2 = 0.284\angle 0° \text{ V}$$

8.4 As shown in the previous problem, the ideal transformer equations are

$$\frac{-\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{2}$$
$$1\mathbf{I}_1 + 2(\mathbf{I}_2) = 0$$

These two equations and the equation for reflecting impedance from the primary of the transformer to the secondary i.e.,

$$\mathbf{Z}_p = \left(\frac{N_1}{N_2}\right)^2 \mathbf{Z}_s$$
$$= \frac{1}{4}\mathbf{Z}_s$$

are the necessary equations for developing the equivalent circuits.

(a) If we reflect the primary to the secondary, we note that

$$\mathbf{V}_2 = -2\mathbf{V}_1$$

And

$$\mathbf{Z}_S = 4\mathbf{Z}_p$$

Therefore, the voltage source in the primary becomes

$$\mathbf{V}_2 = -2(1\angle 0°)$$
$$= 2\angle 180° \text{V}$$

And

$$\mathbf{Z_S} = 4(1)$$
$$= 4\Omega$$

Therefore, the equivalent circuit in this case is shown in Fig. S8.4(a).

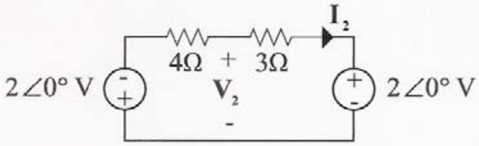

Fig. S8.4(a)

(b) Once again, using the ideal transformer equation to reflect the secondary to the primary we obtain the network in Fig. S8.4(b).

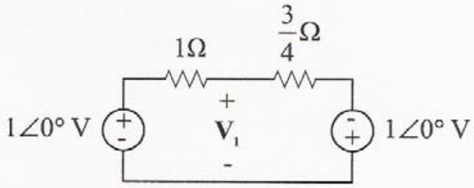

Fig. S8.4(b)