CHAPTER 9 PROBLEMS

9.1 Determine the average power supplied by each source in the circuit in Fig. 9.1.

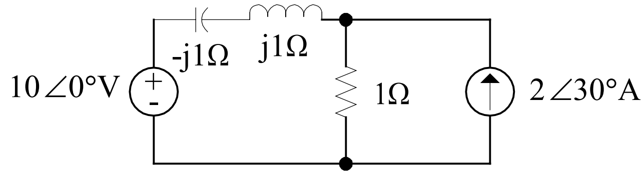

Fig. 9.1

9.2 Given the circuit in Fig. 9.2, determine the impedance $\mathbf{Z_L}$ for maximum average power transfer and the value of the maximum average power transferred to this load.

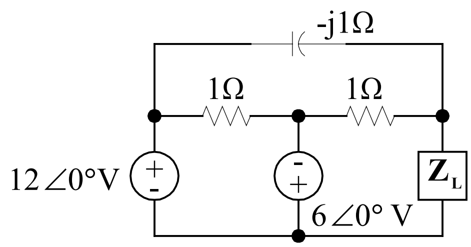

Fig. 9.2

9.3 Calculate the rms value of the waveform shown in Fig. 9.3.

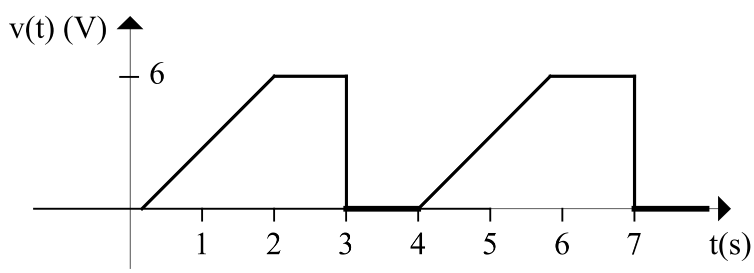

Fig. 9.3

9.4 Determine the source voltage in the network shown in Fig. 9.4.

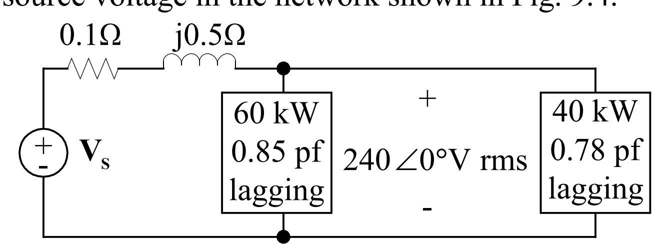

Fig. 9.4

9.5 A plant consumes 75 kW at a power factor of 0.70 lagging from a 240-V rms 60 Hz line. Determine the value of the capacitor that when placed in parallel with the load will change the load power factor to 0.9 lagging.

CHAPTER 9 SOLUTIONS

9.1 Because the series impedance of the inductor and capacitor are equal in magnitude and opposite in sign, from the standpoint of calculating average power the network can be reduced to that shown in Fig. S9.1.

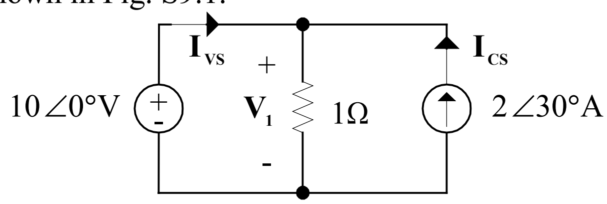

Fig. S9.1

The general expression for average power is

$$P = \frac{1}{2} VI \cos(\theta_V - \theta_I)$$

In the case of the current source $V_1 = 10V$, $I_{CS} = 2A$, $\theta_V = 0°$ and $\theta_I = 30°$. Therefore, the average power delivered by the current source is

$$P_{CS} = \left(\frac{1}{2}\right)(10)(2)\cos(-30°)$$
$$= 8.66 \text{ W}$$

In order to calculate the average power delivered by the voltage source, we need the current I_{VS}. Using KCL

$$\mathbf{I}_{VS} + 2\angle 30° = \frac{\mathbf{V}_1}{1} = 10\angle 0°$$

or

$$\mathbf{I}_{VS} = 8.33\angle\text{-}6.9°\text{A}$$

Now

$$P_{VS} = \frac{1}{2}(10)(8.33)\cos(0° - (-6.9°))$$
$$= 41.34 \text{ W}$$

Therefore, the total power generated in the network is

$$P_T = P_{CS} + P_{VS}$$
$$= 50 \text{ W}$$

Let us now calculate the average power absorbed by the resistor. We know that the average power absorbed by the resistor must be

$$P_R = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\left(\frac{10^2}{1}\right) = 50 \text{ W}$$

In addition, the average power absorbed by the resistor can also be determined by

$$P_R = \frac{1}{2} I_m^2 R$$

However, we do not know the current in the resistor. Using KCL.

$$\mathbf{I_R} = \mathbf{I_{VS}} + \mathbf{I_{CS}} = 8.66\angle{-6.9°} + 2\angle 30° = 10\angle 0° \text{ A}$$

Now

$$P_R = \frac{1}{2}(10)^2(1) = 50 \text{ W}$$

Thus, we find that the total average power generated is equal to the average power absorbed.

9.2 We will first determine the Thevenin equivalent circuit for the network without the load attached. The open-circuit voltage, V_{0C}, can be determined from the network in Fig. S9.2(a).

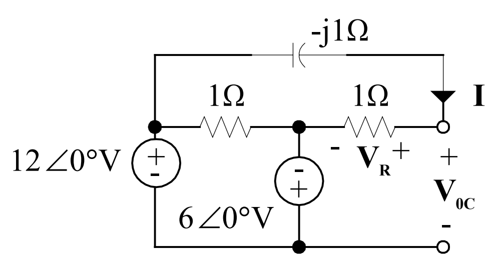

Fig. S9.2(a)

This open-circuit voltage can be calculated in a number of ways. For example, we can compute the current **I** as

$$\mathbf{I} = \frac{12\left(0° - \left(-6\angle 0°\right)\right)}{1 - j} = \frac{18}{1 - j}\ \text{A}$$

Then using KVL,

$$\begin{aligned}\mathbf{V}_{0C} &= 1\mathbf{I} - 6\angle 0° \\ &= \frac{12 + 6j}{1 - j}\ \text{V}\end{aligned}$$

or, we could use voltage division to determine the voltage across the 1Ω resistor on the right, i.e.,

$$\begin{aligned}\mathbf{V}_{R} &= \left[12\angle 0° - \left(-6\angle 0°\right)\right]\left(\frac{1}{1 - j}\right) \\ &= \frac{18}{1 - j}\ \text{V}\end{aligned}$$

Then, once again

$$\begin{aligned}\mathbf{V}_{0C} &= \mathbf{V}_{R} - 6\angle 0° \\ &= \frac{12 + 6j}{1 - j}\ \text{V} \\ &= 9.49\angle 71.56°\text{V}\end{aligned}$$

The Thevenin equivalent impedance is obtained by looking into the open-circuit terminals with all sources made zero. In this case, we replace the voltage sources with short circuits. This network is shown in Fig. S9.2(b).

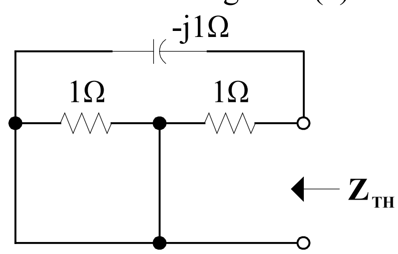

Fig. S9.2(b)

Note that the 1Ω resistor on the left is shorted and thus the $\mathbf{Z}_{TH}$ is

$$\begin{aligned}\mathbf{Z}_{TH} &= \frac{(1)(-j)}{1 - j} = \frac{-j}{1 - j}\ \Omega \\ &= \frac{1}{2} - j\frac{1}{2}\ \Omega\end{aligned}$$

Hence, for maximum average power transfer

$$\mathbf{Z}_L = \mathbf{Z}^*_{TH}$$

or

$$\mathbf{Z}_L = \frac{1}{2} + j\frac{1}{2}\,\Omega$$

Therefore, the network is reduced to that shown in Fig. S9.2(c).

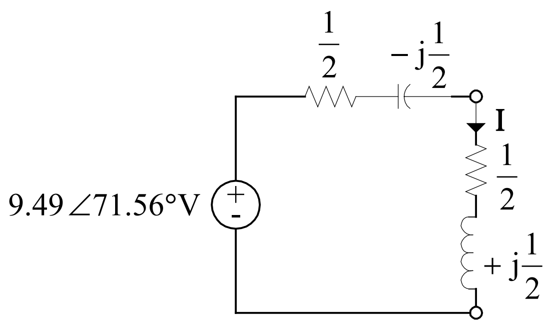

Fig. S9.2(c)

Then

$$\mathbf{I} = \frac{9.49\angle 71.56°}{\frac{1}{2} - j\frac{1}{2} + \frac{1}{2} + j\frac{1}{2}}$$

$$= 9.49\angle 71.56°\ \text{A}$$

and the maximum average power transferred to the load is

$$P_L = \frac{1}{2}(9.49)^2\left(\frac{1}{2}\right)$$
$$= 90\ \text{W}$$

9.3 In order to calculate the rms value of the waveform, we need the equations for the waveform within each of the distinctive intervals.

In the interval $0 \le t \le 2s$, the waveform is a straight line that passes through the origin of the graph. The equation for a straight line in this graph is

$$v(t) = mt + b$$

Where m is the slope of the line and b is the v(t) intercept. Since the line goes through the origin, b = 0. The slope m is

$$m = \frac{6V}{2s} = 3$$

Therefore, in the interval $0 \le t \le 2s$,

$$v(t) = 3t$$

The waveform has constant values in the intervals $2 \le t \le 3s$ and $3 \le t \le 4s$, i.e.,

$$v(t) = 6 \qquad 2 \le t \le 3s$$
$$v(t) = 0 \qquad 3 \le t \le 4s$$

Since the waveform repeats after 4s, the period of the waveform is

$$T = 4s$$

Now that the data for the waveform is known,

$$V_{rms} = \left[\frac{1}{T}\int_0^4 v^2(t)\,dt\right]^{\frac{1}{2}}$$

Therefore, in this case

$$\begin{aligned} V_{rms} &= \left[\frac{1}{4}\left[\int_0^2 (3t)^2\,dt + \int_2^3 (6)^2\,dt + \int_3^4 (0)^2\,dt\right]\right]^{\frac{1}{2}} \\ &= \left[\frac{1}{4}\left[3t^3\Big|_0^2 + 36t\Big|_2^3\right]\right]^{\frac{1}{2}} \\ &= \left[\frac{1}{4}(24 + 36)\right]^{\frac{1}{2}} \\ &= (15)^{\frac{1}{2}} \\ &= 3.87 \text{ V rms} \end{aligned}$$

9.4 We begin our analysis by labeling the various currents and voltages in the circuit as shown in Fig. S9.4.

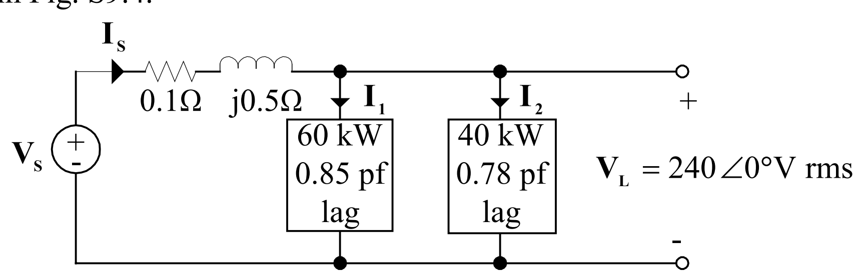

Fig. S9.4

Our approach to determining $\mathbf{V_S}$ is straight forward: We will compute the currents $\mathbf{I_1}$ and $\mathbf{I_2}$; add them using KCL to find $\mathbf{I_S}$; determine the voltage across the line impedance and finally use KVL to add the line voltage and load voltage to determine the source voltage.

The magnitude of the current $\mathbf{I_1}$ is

$$\begin{aligned} |\mathbf{I}_1| &= \frac{P_1}{|\mathbf{V}_L|(pf_1)} \\ &= \frac{60{,}000}{(240)(0.85)} \\ &= 294.12 \text{ A rms.} \end{aligned}$$

And the phase angle is

$$\begin{aligned} \theta_{I_1} &= -\cos^{-1}(0.85) \\ &= -31.79° \end{aligned}$$

The negative sign is a result of the fact that the power factor is lagging.

Thus

$$\mathbf{I_1} = 294.12\angle\text{-}31.79° \text{ A rms.}$$

The magnitude of the current $\mathbf{I_2}$ is

$$\begin{aligned} |\mathbf{I}_2| &= \frac{P_2}{|\mathbf{V}_L|(pf_2)} \\ &= \frac{40{,}000}{(240)(0.78)} \\ &= 213.68 \text{ A rms.} \end{aligned}$$

And the phase angle is

$$\begin{aligned} \theta_{I_2} &= -\cos^{-1}(0.78) \\ &= -38.74° \end{aligned}$$

Thus

$$\mathbf{I_2} = 213.68\angle\text{-}38.74° \text{ A rms.}$$

Using KCL

$$\begin{aligned}\mathbf{I}_S &= \mathbf{I}_1 + \mathbf{I}_2 \\ &= 294.12\angle -31.79° + 213.68\angle -38.74° \\ &= 504.1\angle -34.25° \text{ A rms.}\end{aligned}$$

Then

$$\begin{aligned}\mathbf{V}_S &= \mathbf{I}_S\,(0.1 + j0.5) + 240\angle 0° \\ &= (504.1\angle -34.25°)(0.51\angle 78.7°) + 240\angle 0° \\ &= 257.04\angle 44.44° + 240\angle 0° \\ &= 460.17\angle 23.02° \text{V rms.}\end{aligned}$$

9.5 Since the original power factor is 0.7 lagging the power factor angle is

$$\begin{aligned}\theta_{OLD} &= \cos^{-1}(0.7) \\ &= 45.57°\end{aligned}$$

Then

$$\begin{aligned}Q_{OLD} &= P_{OLD} \tan \theta_{OLD} \\ &= 75{,}000 \tan 45.57° \\ &= 76.52 \text{ kvar}\end{aligned}$$

Hence

$$\begin{aligned}S_{OLD} &= 75{,}000 + j76{,}515 \\ &= 107.14\angle 45.57° \text{ kVA}\end{aligned}$$

The new power factor angle we wish to achieve is

$$\begin{aligned}\theta_{NEW} &= \cos^{-1}(\text{new power factor}) \\ &= \cos^{-1}(0.9) \\ &= 25.84°\end{aligned}$$

Then

$$\begin{aligned}Q_{NEW} &= P_{OLD} \tan \theta_{NEW} \\ &= 75{,}000 \tan 25.84° \\ &= 36{,}324 \text{ kvar}\end{aligned}$$

Now the difference between Q_{NEW} and Q_{OLD} is achieved by the capacitor, i.e.,

$$\begin{aligned}Q_{CAP} &= Q_{NEW} - Q_{OLD} \\ &= 36{,}324 - 76{,}515\end{aligned}$$

$$= -40{,}191 \text{ kvar}$$

And since

$$Q_{CAP} = -\omega CV^2$$

Then

$$C = \frac{40{,}191}{(377)(240)^2}$$
$$= 1850.8\,\mu F$$