

Quantum Mechanics for Engineers Assignment #7

1. Consider an arbitrary potential localized on a finite part of the x -axis. The solutions of the Schrödinger equation to the left and to the right of the potential region are given by

$$Ae^{ikx} + Be^{-ikx} \quad \text{---} \quad Ce^{ikx} + De^{-ikx} \rightarrow x$$

respectively. Show that if we write

$$\begin{aligned} C &= S_{11}A + S_{12}D \\ B &= S_{21}A + S_{22}D \end{aligned}$$

that is, relate the "outgoing" waves to the "ingoing" waves by

$$\begin{bmatrix} C \\ B \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix}$$

that the following relations hold

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 &= 1 \\ |S_{12}|^2 + |S_{22}|^2 &= 1 \\ S_{11}S_{12}^* + S_{21}S_{22}^* &= 0 \end{aligned}$$

This is equivalent to the statement that the matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

is unitary. (*Hint:* Use flux conservation and the possibility that A and D are arbitrary complex numbers. The scattering matrix approach is more stable than the transfer matrix approach and is used more in practice.)

2. Using the rules for calculation the transmission matrices and reformulating the problem calculate the elements of the scattering matrix S_{11} , S_{12} , S_{21} and S_{22} for the potential

$$V(x) = \begin{cases} 0, & x < -a \\ V_0, & -a < x < a \\ 0 & x > a \end{cases}$$

and show that the general conditions proved in problem 1 are indeed satisfied. Assume that $m=6E-32$ kg, $V_0=0.8$ eV and $a=3$ nm.

3. Consider the Kronig-Penney potential with $\lambda=3\pi$.

- (a) Make a detailed plot of $\cos x + \frac{\lambda}{2} \frac{\sin x}{x}$ as a function of $x = ka$.
- (b) Show that forbidden energy bands start just above $ka = n\pi$.
- (c) Show that the allowed energy bands get narrower as λ increases.

(d) Plot the energy as a function of q , where $\cos(qa) = \cos x + \frac{\lambda \sin x}{2x}$.