EEE434/591: QUANTUM MECHANICS

1. Starting from the time-dependent SWE:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \ ,$$

that has a formal solution of the form:

$$\psi(t) = \exp\left(-\frac{iHt}{\hbar} \right) \psi(0) \ ,$$

calculate the time variation of the creation and annihilation operators used to describe the 1D harmonic oscillator problem. In your calculations, use:

$$A_H(t) = e^{iHt/\hbar} A_S e^{-iHt/\hbar} \quad ,$$

where A_S is some arbitrary operator in the Schrödinger picture, and $A_H(t)$ is the representation of this operator in the Heisenberg picture.

2. Using Fermi's Golden rule described in class, calculate the total scattering rate out of state k for the case of a delta-function perturbing potential, i.e. $V(\mathbf{r}) = V_0 \delta(\mathbf{r})$. In your calculations consider a three-dimensional crystal and plane-waves as basis functions.

3. The matrix element for scattering between states **k** and **k**' for polar optical phonon scattering, which is dominant scattering mechanism for GaAs bulk material at low electric fields, equals to

$$\left| M(\mathbf{k}, \mathbf{k}') \right|^2 = \frac{\hbar e^2 \omega_{LO}}{2Vq^2} \left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon(0)} \right) \begin{bmatrix} N_0 \\ N_0 + 1 \end{bmatrix} \delta(\mathbf{k} - \mathbf{k}' \pm \mathbf{q})$$

where:

k = wavevector corresponding to the initial state,

k' = wavevector corresponding to the final state,

q = momentum transfer in the scattering process,

N_0 = phonons occupancy factor given by the Bose-Einstein statistics (top term in the square brackets corresponds to the phonon absorption and the bottom to the phonon emission process),

$\varepsilon_\infty = 10.92 \cdot \varepsilon_0$ is the high-frequency dielectric permittivity,

$\varepsilon(0) = 12.9 \cdot \varepsilon_0$ is the low-frequency dielectric permittivity, and

$\hbar\omega_{LO} = 0.03536$ [eV] is the energy of the longitudinal optical phonons.

Calculate and plot the energy dependence of the:

(a) Total scattering rate out of an initial state **k** defined as

$$\frac{1}{\tau(k)} = \sum_{\mathbf{k}'} S(\mathbf{k},\mathbf{k}'),$$

(b) Momentum relaxation rate defined as

$$\frac{1}{\tau_m(k)} = \sum_{\mathbf{k}'} \left(1 - \frac{\mathbf{k}\cdot\mathbf{k}'}{k^2}\right) S(\mathbf{k},\mathbf{k}'),$$

(c) Energy relaxation rate, for which you need to use

$$\frac{1}{\tau_E(k)} = \sum_{\mathbf{k}'} \left(1 - \frac{E(\mathbf{k}')}{E(\mathbf{k})}\right) S(\mathbf{k},\mathbf{k}').$$

In the above expressions, $S(\mathbf{k},\mathbf{k}')$ equals the transition rate from some initial state **k** to final state **k'**, that is calculated using the Fermi's Golden Rule derived in class.