

**DESCRIPTION OF VARIOUS EXPERIMENTALLY  
MEASURED MOBILITIES**

Electron mobility in a surface inversion and accumulation layers is an important parameter which must be accurately known when modeling MOS devices. In particular, the variation of this parameter with substrate doping, vertical electric field, crystal orientation, oxide and interface charges, surface-roughness and temperature must be quantitatively known if MOS transistor models are to correctly simulate actual devices. In general, three different types of scattering mechanisms account for the mobility behavior in MOS devices, when the gate voltage is above threshold:

- Phonon scattering due to the various modes of lattice vibrations including both acoustic and nonpolar optical phonons. This scattering mechanism is important at room temperature and can be ignored at very low temperatures.
- Coulomb scattering due to charged centers in the oxide, at the interface and due to ionized impurities. Coulomb scattering is important for lightly inverted surfaces. High interface-trap densities or substrate doping concentrations imply high Coulomb scattering. This dissipative process is less effective for heavily inverted surfaces due to the increased carrier screening.
- Surface-roughness scattering, that arises because of the deviation of the interface from an ideal plane. This type of scattering is important under strong inversion conditions because the strength of the interaction is governed by the distance of the carriers from the interface; the closer the carriers to the interface, the stronger the scattering due to surface-roughness.

The relative importance of the various scattering mechanisms depends upon the temperature and the strength of the surface electric field, which determines the inversion charge density. At low temperatures, mobility is limited by Coulomb and surface-roughness scattering only. At room temperature, it is governed by Coulomb scattering in

the low-field region and it is dominated by surface-roughness and phonon scattering under strong inversion.

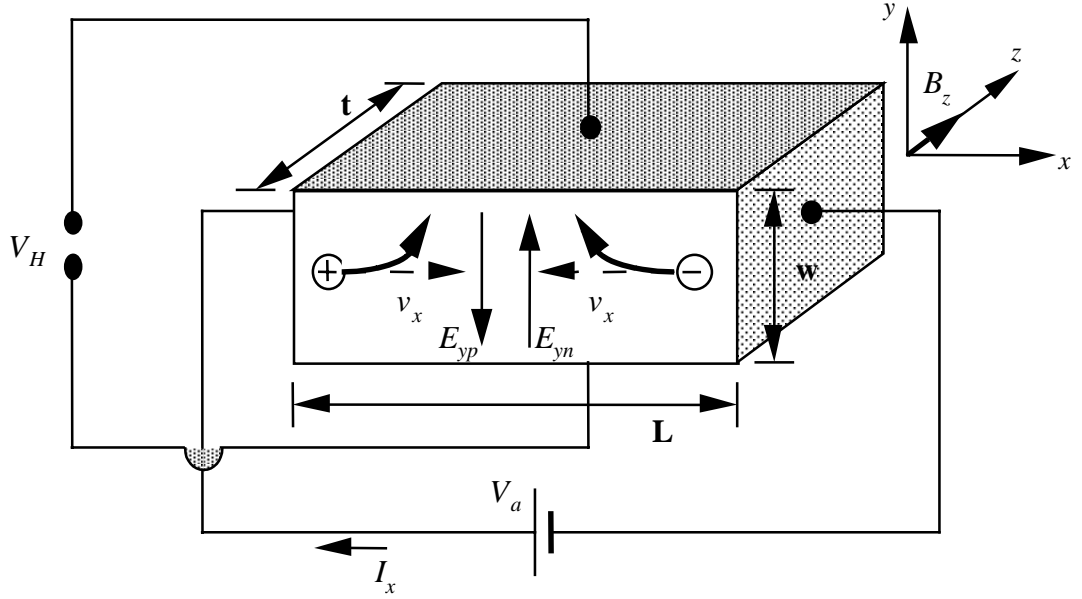
In the early days, most experimental work on inversion layer mobilities has concentrated on *Hall* and *field-effect* mobilities. However, it is the *effective* mobility which appears in all theoretical models of MOS transistors and which is, therefore, most useful in modern MOS device modeling. Of lesser importance is the so-called *saturation* mobility. The Hall mobility, described in section B.1, represents the bulk mobility and the interface, as well as the quantization effect, plays a minor role in its determination. The field-effect, effective and saturation mobilities, used to characterize MOSFET's, are described in section B.2.

### **B1. Hall mobility**

The Hall measurement technique is commonly used for resistivity measurements, carrier concentration characterization as well as mobility measurements. The basic setup of the Hall technique is given in Fig. B.1. As shown in the figure, the applied electric field along the  $x$ -axis gives rise to a current  $I_x$ . The Lorentz force  $F_y = ev_x B_z$  due to the applied magnetic field along the positive  $z$ -axis pushes the carriers upwards. This results in a pile up of electrons and holes at the top part of the sample which, in turn, gives rise to electric fields  $E_{yn}$  and  $E_{yp}$ , respectively. The transverse electric fields along the  $y$ -axis are called *Hall fields*. Since there is no net current along the  $y$ -direction in steady-state, the induced electric fields along the  $y$ -axis exactly balance the Lorentz force, i.e.

$$\frac{V_H}{w} = R_H J_x B_z . \quad (\text{B-1})$$

In (B-1),  $J_x$  is the current density and  $R_H$  is the so-called Hall coefficient. If both



**Figure B.1** Experimental setup for Hall measurement technique.

electrons and holes are present in the sample, the Hall coefficient is given by

$$R_H = \frac{r_h p - r_e b^2 n}{e(p + bn)^2}, \quad (\text{B-2})$$

where  $n$  ( $p$ ) is the electron (hole) concentration,  $b = \mu_e/\mu_h$  is the mobility ratio and  $r_e$  ( $r_h$ ) is the so-called Hall scattering factor for electrons (holes) that takes into account the energy spread of the carriers. The Hall scattering factor that appears in (B-2), is defined by the ratio

$$r = \frac{\langle \tau^2 \rangle}{\langle \tau \rangle^2}, \quad (\text{B-3})$$

where  $\tau$  is the mean-free time between carrier collisions, and the average value of the  $m$ th power of  $\tau$  in  $d$ -dimensions is calculated from

$$\langle \tau^m \rangle = \frac{\int_0^\infty \epsilon^{d/2} \tau^m(\epsilon) (\partial f_0 / \partial \epsilon) d\epsilon}{\int_0^\infty \epsilon^{d/2} (\partial f_0 / \partial \epsilon) d\epsilon}, \quad (\text{B-4})$$

where  $f_0$  is the equilibrium Fermi-Dirac distribution function.

The Hall mobility  $\mu_H$  is defined as a product of the Hall coefficient  $R_H$  and conductivity  $\sigma_x$

$$\mu_H = |R_H| \sigma_x, \quad (\text{B-5})$$

which is calculated from

$$\sigma_x = \frac{I_x L}{wt V_a}. \quad (\text{B-6})$$

It is important to point out that the Hall mobility has to be distinguished from the so-called conductivity (or effective) mobility which does not contain the Hall scattering factor. The two mobilities are related to each other according to

$$\mu_H = r \mu_{eff}. \quad (\text{B-7})$$

In conclusion, since the Hall scattering factor is generally larger than unity, the Hall mobility can differ significantly from the conductivity mobility. In that respect, the Hall-effect measurements cannot give the mobility unambiguously without an adequate theory of scattering.

## B.2 MOSFET mobilities

Electron mobility in surface-inversion layers has been of considerable interest for many years. At present, several mobilities (already mentioned in the introduction part of this appendix) are used to characterize MOSFETs.

The *effective mobility*  $\mu_{eff}$  is usually deduced from the first-order one-dimensional model in the linear mode. At low drain voltages ( $V_{DS} = 10 - 50mV$ , where  $V_{DS} \ll V_{GS} - V_T$ ), the effective mobility is related to the *drain conductance*

$$g_D = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS} = \text{const.}}, \quad (B-8) \quad 6$$

according to

$$\mu_{eff} \approx \frac{Lg_D}{ZC_{ox}(V_{GS} - V_T)}. \quad (B-9)$$

In expressions (B-8) and (B-9),  $I_D$  is the drain current,  $L$  is the length and  $Z$  is the width of the channel,  $V_{GS}$  is the gate voltage, and  $V_T$  is the so-called threshold gate voltage. The threshold gate voltage is often defined as the voltage where the Fermi level is as close to the conduction (or valence) band at the surface as to the valence (or conduction) band in the bulk. It is experimentally determined by using various linear extrapolation techniques on the  $I_D - V_{GS}$  curves. The inaccuracies in the threshold voltage significantly affect the effective mobility results. Both thermal broadening and trapping tend to obscure the accurate measurements of  $V_T$  and, therefore,  $\mu_{eff}$ .

The previously described effective mobility is distinct from the so-called *field-effect mobility*  $\mu_{FE}$  which is obtained from the MOSFET *transconductance*

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} = \text{const.}} \quad (B-10)$$

through the expression

$$\mu_{FE} = \frac{Lg_m}{ZC_{ox}V_{DS}}. \quad (B-11)$$

The experimentally measured field-effect mobility is usually smaller than the effective mobility. The discrepancy between the effective and field-effect mobility is associated with the neglect of the electric-field dependence (more precisely, the neglect of the gate voltage dependence) in the derivation of the expression for  $\mu_{FE}$ . For example, for the device in the linear regime and using the definitions given in (B-8) and (B-10), after a straightforward calculation it follows that the two mobilities can be related to each other according to

$$\mu_{FE} \approx \mu_{eff} + (V_{GS} - V_T) \left. \frac{\partial \mu_{eff}}{\partial V_{GS}} \right|_{V_{DS}=const.} . \quad (\text{B-12})$$

Since the effective mobility decreases with the gate voltage, i.e.  $\partial \mu_{eff} / \partial V_{GS} < 0$  (except for very low gate voltages, where it actually increases due to the decreased importance of Coulomb scattering),  $\mu_{FE} < \mu_{eff}$ . Therefore, if  $\mu_{FE}$  is used for device modeling, the currents and device switching speeds are going to be underestimated.

Very rarely, the MOSFET mobility is obtained from the output current-voltage characteristics with the device in saturation. In this regime, the saturation drain current  $I_{D,sat}$  is calculated from

$$I_{D,sat} = \frac{BZ\mu_{sat}C_{ox}}{2L} (V_{GS} - V_T)^2, \quad (\text{B-13})$$

where  $B$  is the body factor, which is not always well known. If one plots the variation of  $\sqrt{I_{D,sat}}$  vs.  $(V_{GS} - V_T)$ , then the so-called *saturation mobility* is determined from the slope  $m$  of this curve, according to

$$\mu_{sat} = \frac{2Lm^2}{BZC_{ox}} . \quad (\text{B-12})$$

Again, due to the neglect of the gate-voltage dependence in the definition for the saturation mobility, the experimental results for  $\mu_{sat}$  are always smaller compared to the ones obtained for  $\mu_{eff}$ .